ENGINEERING THERMODYNAMICS

For III Semester B.E., Mechanical Engineering Students

As per Latest Syllabus of Anna University - TN

With Short Questions & Answers and University Solved Papers

New Regulations 2017

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ME8391 ENGINEERING THERMODYNAMICS

UNIT I: BASIC CONCEPTS AND FIRST LAW 9 + 6

UNIT II: SECOND LAW AND AVAILABILITY ANALYSIS 9 + 6

UNIT III: PROPERTIES OF PURE SUBSTANCE AND STEAM POWER CYCLE 9 + 6

UNIT IV: IDEAL AND REAL GASES, THERMODYNAMIC RELATIONS 9 + 6

UNIT V: GAS MIXTURES AND PSYCHROMETRY 9 + 6
Mole and Mass fraction, Dalton’s and Amagat’s Law. Properties of gas mixture – Molar mass, gas constant, density, change in internal energy, enthalpy, entropy and Gibbs function. Psychrometric properties, Psychrometric charts. Property calculations of air vapour mixtures by using chart and expressions. Psychrometric process – adiabatic saturation, sensible heating and cooling, humidification, dehumidification, evaporative cooling and adiabatic mixing. Simple Applications

TOTAL: 75 PERIODS
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UNIT - I

Basic Concepts and First Law

1.1 INTRODUCTION

Thermodynamics can be defined as the science which deals with the transformation of energy and its effects on the properties of the substance. The word ‘Thermodynamics’ originates from two Greek words, ‘therme’ (heat) and ‘dynamis’ (motion or power). Thermodynamics can also be called as the science of 3E’s i.e., Equilibrium, Energy and Entropy.

Thermodynamics is a branch of Science that deals with the relationship among heat, work and properties of system which are in equilibrium with one another.

One of the most fundamental laws of nature is the conservation of energy principle. It states that during an interaction, energy can change from one form to another but the total amount of energy remains constant. i.e., Energy can neither be created nor be destroyed, but it can be transformed from one form to another. For example,

1. A ball falling from a tower picks speed as a result of its potential energy being converted to kinetic energy.

2. A person eating more and working less i.e., a person who has a greater energy input (food) than energy output (exercise) will gain weight (store energy in the form of fat)

The first law of thermodynamics is based on the conservation of energy principle. The second law of thermodynamics reveals that energy has quality as well as quantity and actual processes occur in the direction of decreasing quality of energy. For example, a cup of hot coffee in a room eventually cools, but a cup of cool coffee never gets hot by itself.

The first and second laws of thermodynamics emerged simultaneously in the 1850s, by the works of James Prescott Joule, William Rankine, Rudolph Clausius and Lord Kelvin.
1.1.1 Role of Thermodynamics in Engineering and Science

Thermodynamics plays a vital role in the field of Engineering and Science. They are explained as follows.

Thermodynamics is very important in the design, analysis and development of Engineering systems such as the power-producing systems like engines, turbines etc. and the power-consuming systems like compressor, pumps, etc.

The working principle of many scientific devices like thermometer, thermistor, thermostat etc... involves thermodynamics.

Good understanding and careful applications of the fundamentals of thermodynamics leads to improved design, increased efficiency, optimum operating conditions and decreased levels of environmental pollution etc.

1.1.2 Applications of Thermodynamics

Thermodynamic principles are applied in various fields of Engineering and Technology. Some of the fields are

(i) Internal combustion engine (ii) Gas turbines
(iii) Air conditioner & Refrigerator (iv) Compressor
(v) Steam and Nuclear power plants (vi) Rockets
(vii) Jet engines etc.

Applications of thermodynamics are found in various aspects of life eg: A commonly encountered thermodynamics application is, ‘the human heart, which is constantly pumping blood to all parts of the body. The movement of blood and various energy conversions generates heat, within the body. (normally called as body heat). The generated heat is constantly rejected to the environment and the process is called as metabolic heat rejection. However, this heat rejection can be controlled by adjusting our clothing to the environment conditions.

Other application of thermodynamics involves increasing the heat transfer rate like exhaust fan in a house, cooling fans in the computer CPU and fins provided on the engine cylinders etc.
### 1.1.3 Founders of Thermodynamics

<table>
<thead>
<tr>
<th>Founders</th>
<th>Contribution</th>
</tr>
</thead>
</table>
| 1. Antoine Laurent Lavoisier (1743 - 1794)    | - Father of Modern Chemistry  
- Developed combustion process  
- Also, proposed the name for ‘oxygen’ (which was discovered by Joseph Priestely) and mentioned its importance in the combustion process. |
| 2. Nicolas Leonard Sadi Carnot (1796 - 1832) | - Father of Thermodynamics  
- Developed Carnot cycle  
- Established the basic principles governing the conversion of heat to work, that led to the fundamental laws of thermodynamics. |
| 3. James Prescott Joule (1818 - 1889)        | - Established ‘heat’ as a form of energy which led to the dismissal of ‘calorific theory of heat’.  
- Developed ‘First law of Thermodynamics’ with experimental proof that supports the heat, is also an energy. |
| 4. William Thompson (also called as Lord Kelvin) (1829 - 1907) | - Absolute temperature scale is named after him.  
- Developed Thermodynamic temperature scale.  
- Along with Joule, he expounded Joule-Kelvin effect. |
| 5. Rudolph J. Clausius (1822 - 1888)         | - Developed ‘Second Law of Thermodynamics’  
- Also, led to the founding of the Thermodynamic property called ‘entropy’. |
1.2 BASIC CONCEPTS

Some important basic concepts of thermodynamics such as system state, equilibrium, path, process, energy, properties and so on are discussed in this Chapter.

1.2.1 Units and Dimensions

Nowadays, the SI (System International) system of units has been used. Any physical quantity can be characterised by dimensions. The arbitrary magnitudes assigned to the dimensions are called Units. Some basic dimensions such as mass $m$, length $L$, time $t$ and temperature $T$ are selected as primary (or) fundamental dimensions, while others such as velocity $V$, energy $E$, and volume $V$ are expressed in terms of primary dimensions and are called secondary dimensions (or) derived dimensions.

A number of unit systems have been developed over the years. But nowadays, the SI (System International) system of units are widely used. The SI is simple and logical system. Some fundamental dimensions and their units in SI are given in the following table.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>metre</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Temperature</td>
<td>Kelvin</td>
<td>K</td>
</tr>
<tr>
<td>Electric current</td>
<td>Ampere</td>
<td>A</td>
</tr>
<tr>
<td>Amount of light</td>
<td>Candela</td>
<td>cd</td>
</tr>
<tr>
<td>Amount of matter</td>
<td>mole</td>
<td>mol</td>
</tr>
</tbody>
</table>

1.2.2 Mass ($m$)

It is quantity of matter. Its unit is ‘kg’.

1.2.3 Force

In SI, the unit for force is derived from Newton’s second law. i.e., Force = (Mass) × (Acceleration)

$$F = m a$$
In SI, the force unit is the Newton (N) and it is defined as the force required to accelerate a mass of 1 kg at a rate of 1 m/s², i.e., $1 \, \text{N} = 1 \, \text{kg} \cdot \text{m/s}^2$.

The weight of a body ($W$) is the force with which the body is attracted to the centre of the earth. It is the product of its mass ($m$) and the local gravitational acceleration ($g$), i.e., $W = mg$

where $g = 9.80665 \, (9.81) \, \text{m/s}^2$ at sea level. The mass ($m$) of a body will remain same regardless of its location in the universe. Its weight, however, will change with a change in gravitational acceleration. A body will weigh less on top of a mountain since $g$ decreases with altitude. At sea level, a mass of 1 kg will weigh 9.807 N.

1.2.4 Pressure

Pressure is the normal force exerted by a system against unit area of the bounding surface.

If $\delta A$ is a small area and $\delta A'$ is the smallest area from the concept of continuum, and $\delta F_n$ is the component of force normal to $\delta A$, the pressure $P$ at a point on the wall is defined as

$$P = \lim_{\delta A \to \delta A'} \frac{\delta F_n}{\delta A}$$

The pressure $P$ at a point in a fluid in equilibrium is the same in all directions.

The unit for pressure in the SI system is the Pascal (Pa), which is the force of 1 N acting on an area of 1 m².

$1 \, \text{Pascal} = 1 \, \text{Pa} = 1 \, \text{N/m}^2$

The unit of pascal is very small. Hence, very often, kilo-pascal (kPa) (or) mega-pascal (MPa) is used. Sometime bar and standard atmosphere are used which are not within the SI system of units.

$1 \, \text{bar} = 10^5 \, \text{Pa} = 100 \, \text{kPa} = 0.1 \, \text{M Pa}$ and

$1 \, \text{Standard atmosphere} = 1 \, \text{atm} = 101.325 \, \text{kPa} = 1.01325 \, \text{bar}$
Most of the instruments indicate pressure relative to the atmospheric pressure whereas the pressure of a system is its pressure above zero (or) relative to a perfect vacuum.

*The pressure relative to the atmosphere is called gauge pressure.*

*The pressure relative to a perfect vacuum is called absolute pressure.*

\[
\text{Absolute pressure} = \text{Gauge pressure} + \text{Atmospheric pressure}
\]

The SI unit prefixes are used in all branches of engineering. Some standard prefixes in SI units for Mechanical Engineering branch are given in the table.

<table>
<thead>
<tr>
<th>Multiple</th>
<th>Prefix</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^{12})</td>
<td>Tera, (T)</td>
</tr>
<tr>
<td>(10^9)</td>
<td>Giga, (G)</td>
</tr>
<tr>
<td>(10^6)</td>
<td>Mega, (M)</td>
</tr>
<tr>
<td>(10^3)</td>
<td>Kilo, (k)</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>milli, (m)</td>
</tr>
<tr>
<td>(10^{-6})</td>
<td>micro, (\mu)</td>
</tr>
<tr>
<td>(10^{-9})</td>
<td>nano, (n)</td>
</tr>
<tr>
<td>(10^{-12})</td>
<td>pico, (p)</td>
</tr>
</tbody>
</table>

1.2.5 Temperature

It is defined as the degree of intensity of heat or measure of hotness or coldness of a body.

Temperature - Intensive property - °C or K (Centigrade or Kelvin).

\[30^\circ C = 30 + 273 = 303 \text{ K}\]

Normally, \(t\) is for °C and \(T\) is for K.

1.2.6 Specific Volume and Density

Volume \((V)\) is the space occupied by a substance and is measured in \(m^3\). The specific volume \((v)\) of a substance is defined as the volume per unit mass and is measured in \(m^3/\text{kg}\). From the concept of continuum, the specific volume at a point is defined as
\[ v = \lim_{\delta V \to \delta V'} \frac{\delta V}{\delta m} \]

where \( \delta V' \) is the smallest volume for which the system can be considered as continuum.

**Specific volume** (\(v\))

- **Volume** = Total volume = \(V\) in \(m^3\)
- Specific volume = \(\frac{\text{Volume}}{\text{mass}} = \frac{V}{m} = v\)

Its unit is \(m^3/\text{kg}\)

**Density** (\(\rho\)):

- It is the mass of the substance per unit volume. Its unit is \(\text{kg/m}^3\).
- Density (\(\rho\)) is the mass per unit volume of a substance and is given in \(\text{kg/m}^3\). \(\rho = \frac{m}{V}\)

### 1.2.7 Energy

Work, which is a form of energy, can simply be defined as force times distance. Therefore, it has the unit “Newton-meter (N-m)” which is called a **Joule** (\(J\)). i.e.,

\[ 1 \text{ J} = 1 \text{ N-} \text{m} \]

A more common unit for energy in SI is the kilojoule (1 kJ = \(10^3\) J). The energy per unit mass is the specific energy and its unit is \(J/\text{kg}\).

In the metric system, the amount of energy needed to raise the temperature of 1 kg of water at 15°C by 1°C is defined as 1 kiloCalorie (kCal), and

\[ 1 \text{ kCal} = 4.187 \text{ kJ}. \]

### 1.2.8 Power

The rate of energy transfer (or) storage is called power. The unit of power is Watts (W), kiloWatts (kW) (or) MegaWatts (MW).

\[ 1 \text{ W} = 1 \text{ J/s} = 1 \text{ N} \cdot \text{m/s}; \quad 1\text{kW} = 1000 \text{ W} \]

### 1.2.9 Dimensional Homogeneity

In engineering, all equations must be **dimensionally homogeneous**, i.e., every term in an equation must have the same unit.
Analyse the following equation, \( E = 35 \text{ kJ} + 8 \text{ kJ/kg} \)
where \( E \) is the total energy in kJ. Since the two terms on the right hand side do not have the same units, they cannot be added to obtain the total energy. Multiplying the last term \((8 \text{ kJ/kg} \times m \text{ kg})\) by mass will eliminate the kilograms in denominator, and the whole equation will become dimensionally homogeneous i.e., every term in the equation will have the same unit.

i.e., \( E = 35 \text{ kJ} + 8 m \text{ kJ} \)

### 1.3 MACROSCOPIC AND MICROSCOPIC APPROACH

Normally, a substance consists of a large number of particles called molecules and the properties of substance naturally depend on the behaviour of these particles. There are two approaches from which the behaviour of particles can be studied. They are

<table>
<thead>
<tr>
<th>Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macroscopic Approach</td>
</tr>
<tr>
<td>(or) Classical Thermodynamics</td>
</tr>
<tr>
<td>Microscopic Approach (or) statistical Thermodynamics</td>
</tr>
</tbody>
</table>

#### 1.3.1 Macroscopic viewpoint

In this point of view, a certain amount of matter is considered, neglecting the events occurring at the molecular level. It does not require a knowledge on the behaviour of individual particles. It provides a direct and easy way to the solution of engineering.

In general, macroscopic point of view is just like, “inventing a device or using an instrument, to measure the quantities directly, instead of finding it out through calculation, derivation and formulas”.

eg: ‘Pressure gauge’ is used to measure pressure directly without calculating force on a given unit area.

Likewise, temperature can also be measured using thermometer.

However, macroscopic point of view is limited to those effects that can be perceived by human senses.

#### 1.3.2 Microscopic viewpoint

*What is microscopic approach in thermodynamics? (Nov/Dec 2013 - AU)*

In this point of view, behaviour of each molecule has to be studied, to find out the behaviour of the entire substance. Each molecule at a given
instant has certain position, velocity and energy, for each molecule these changes frequently as a result of collisions. And also, these effects cannot be perceived by human senses.

In general, microscopic point of view requires large number of data and equations to find out the quantities through calculation and derivation.

eg: Impulse, Enthalpy, Entropy, Gibbs function, Helmhotz function etc.

Table: Comparison of Microscopic and Macroscopic approach

<table>
<thead>
<tr>
<th>Macroscopic approach</th>
<th>Microscopic approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. It is also called as classical thermodynamics.</td>
<td>It is also called as statistical thermodynamics.</td>
</tr>
<tr>
<td>2. Behaviour of each molecule need not to be determined.</td>
<td>Behaviour of each molecule need to be determined.</td>
</tr>
<tr>
<td>3. Molecular effects can be perceived by human senses.</td>
<td>Molecular effects cannot be perceived by human senses.</td>
</tr>
<tr>
<td>4. Classical thermodynamics, however can be derived from statistical thermodynamics.</td>
<td>Statistical thermodynamics cannot be determined using classical thermodynamics.</td>
</tr>
<tr>
<td>5. eg: Pressure, Temperature</td>
<td>eg: Enthalpy, Entropy</td>
</tr>
</tbody>
</table>

1.4 CONCEPT OF CONTINUUM

Define the term Continuum

Generally, matter is made up of atoms & these atoms are widely spaced in the gas phase. From the macroscopic viewpoint, behaviour of individual molecules can be disregarded and the matter with widely spaced atoms can be treated as continuous because a large no. of molecules can accommodate even in an extremely small volume of atom. This view of continuous, homogeneous matter with no holes is called as “continuum”.

The continuum idealization allow us to treat properties as a point function and to assume that the properties vary continuously in space with no jump discontinuities.

Concept of continuum can be verified by the dimensionless parameter known as knudsen number ($k_n$).
If Knudsen number, $k_n < 0.01$, then the concept of continuum holds good, i.e., the continuum model can be applicable as long as the characteristic length of the system is much larger than the mean free path of the molecules.

At very high vacuums or high elevations, the mean free path becomes large, for such cases “rarefield gas flow theory” should be used and the impact of individual molecules should be considered.

But in most engineering applications, however the continuum is valid and convenient & goes hand in hand with the macroscopic point of view.

1.5 THERMODYNAMIC SYSTEMS

Thermodynamic system is known as a space or constrained area upon which our attention is concentrated on. It is the region to be studied. The system has boundary (or) boundary line. Anything outside the boundary is called surroundings. In other words, the system and surroundings are separated by boundary. (Boundary may be real or imaginary).

1.5.1 Types of system

The systems are classified into:

1. Closed system, 2. Open system, 3. Isolated system.

System may be considered to be closed (or) open, depending on whether a fixed mass (or) a fixed volume in space is chosen for study.

1.6 CLOSED SYSTEM

A closed system, also known as a control mass, consists of a fixed amount of mass and no mass can cross its boundary.

i.e., no mass can enter (or) leave a closed system as shown in Fig. 1.3 But energy, in the form of heat (or) work, can cross the boundary. And the volume of a closed system does not have to be fixed.
Energy is transferred from the surroundings into the system and from the system to the surroundings. But there is no mass transfer in closed system.

Eg. Piston and cylinder without valves Fig. 1.4.

1.7 OPEN SYSTEM

An open system is a properly selected region in space. It usually encloses a device that involves mass flow. A large number of engineering problems involve mass flow in and out of a system and, therefore, are modeled as control volumes. In general, any arbitrary region in space can be selected as a control volume. The boundaries of a control volume are called a control surface as shown in Fig. 1.6 and they can be real or imaginary.

Both energy and mass are transferred from the system to surroundings and from the surroundings to system.

(e.g) 1. Air compressor (Fig 1.5), 2. I.C. Engines, 3. Turbines, 4. Boilers, 5. Centrifugal pumps, 6. Nozzles.

Most of the engineering devices are open systems.
1.8 ISOLATED SYSTEM

Neither energy nor mass transfer occurs between the system and surroundings (Fig 1.7).

Practically, no system is isolated.

1.9 HOMOGENEOUS AND HETEROGENEOUS SYSTEMS

A system, which contains only a single phase, is called as Homogeneous system. Eg. Mixture of air and water vapour.

A system, which consists of more than one phase, is called as Heterogeneous system. Eg. Mixture of water and steam.

1.9.1 Pure substance

A Substance with homogeneous and invariable chemical composition even though change of phase takes place is called as pure substance. Eg. Water.

1.9.2 State postulate

The number of independent intensive properties are required to fix the state of the system is given by state postulate.

For a pure substance, this is called as two property rule. i.e., two intensive properties are required to fix the state of the pure substance.

Number of independent intensive properties can be found analytically using Gibb’s rule

\[ P + F = C + 2 \]

\( P \) - No. of phase, \( C \) - No. of components
\( F \) - No. of independent intensive properties

eg: For single liquid (or) single gas

\( P = 1, \quad C = 1 \)

\[ \therefore F = C - P + 2 = 1 - 1 + 2 = 2 \]
1.10 PROPERTY

What are intensive and extensive properties?

(Nov/Dec 2016 - AU)

It is a characteristic of the system. The system is identified by some quantities like temperature, pressure, volume etc.

1. **Intensive Property**: The properties which do not depend on the mass of the system, are called intensive property.

   *Intensive property* - Independent of the mass of the system: Example - Temperature, pressure, density, surface tension, specific gravity, etc.

2. **Extensive Property**: Those properties which depend on the mass of the system are called extensive property.

   *Example*: Volume, Force, eg: electric charge, weight, energy (heat and work) magnetization. If mass is more, then volume will be more. (Fig 1.8).

Classify the following properties as intensive or extensive or neither
(a) Pressure (b) Temperature (c) Volume (d) Internal energy (e) Volume per mole (f) Mass (g) Enthalpy per unit mass.  
(Nov/Dec 2015 - AU)

1.10.1 Total and specific quantities

The extensive property divided by mass will become intensive property.

(e.g) Volume (m$^3$)/mass = Specific volume m$^3$/kg

(Intensive)

Enthalpy (kJ)/mass = Specific enthalpy kJ/kg

(Intensive)

Here volume is total quantity and specific volume is specific quantity.

Similarly, enthalpy is total quantity and specific enthalpy is specific quantity.

Generally, upper case letters are used to denote extensive properties (with mass ‘m’ being a major exception), i.e total quantities (and lowercase letters are used for intensive properties i.e specific quantities. (with pressure $P$ and temperature $T$ being the obvious exceptions).
1.11 THERMODYNAMIC EQUILIBRIUM

Define thermodynamic equilibrium (Nov/Dec 2014 - AU)

When mechanical equilibrium, chemical equilibrium and thermal equilibrium are satisfied, then the system is said to be in thermodynamic equilibrium.

When any property remains same with respect to time, it is called as equilibrium or steady state.

Eg. Temperature and pressure at all points of a system are same means that the system exists in thermodynamic equilibrium.

1. Mechanical Equilibrium

If the system’s pressure remains same with respect to time, then the system is said to be in mechanical equilibrium.

2. Chemical Equilibrium

If the system’s chemical composition remains same with respect to time, then the system is said to be in chemical equilibrium.

3. Thermal Equilibrium

If the system’s temperature remains same with respect to time, then the system is said to be in thermal equilibrium.

1.12 STATE, PATH AND PROCESS


Consider a system does not undergoing any change. At this point, all the properties can be measured (or) calculated throughout the entire system, which gives us a set of properties that completely describes the condition, or the state, of the system. At a given state, all the properties of a system have fixed values. If the value of even one property changes, the state will change to a different one. A system is shown at different states in the Fig. 1.9.

Thermodynamics deals with equilibrium state. The word equilibrium implies a state of balance. In an equilibrium state, there are no unbalanced potentials (or driving forces) within the system.
There are many types of equilibrium, and a system is not in thermodynamic equilibrium unless the conditions of all the relevant types of equilibrium are satisfied.

For example, a system is in **thermal equilibrium** if the temperature is same throughout the entire system, as shown in Fig. 1.10.

That is, the system involves no temperature differential, which is the driving force for heat flow.

Any change that a system undergoes from one equilibrium state to another is called a **process**, and the series of states through which a system passes during a process is called the **path** of the process. To describe a process completely, one should specify the initial and final states of the process, as well as the path it follows and the interactions with the surroundings.

Refer the Fig. 1.11 (a) showing Pressure - volume (ie) PV diagram and Fig. 1.11 (b) showing piston - cylinder arrangement.

The **state** is a condition of the system at particular time. Each and every condition of the system is called **state**.

At state (1), the pressure is $P_1$ and the volume is $V_1$. Similarly at state (2), the pressure is $P_2$ the volume is $V_2$. In between 1 and 2, there are so many states $A$, $B$, $C$, ..., $G$.

So every time the system is under the change of states. If all the states are traced, it is the **path** of the change of states. If the path is completely specified, the change of state is called **process**. The example given is a compression process.
1.13 CYCLE

Explain the following terms. (a) Process (b) Cycle  (Nov/Dec 2011 - AU)

The series of processes whose end states are identical, then it is called cycle. Refer Fig 1.12.

1–2 = The process A
2–1 = The process B
1–2–1 = Cycle

The process A and the process B form a cycle. Because initial state is 1 and final state is also 1. So it is a cycle (or) thermodynamic cycle.

A system is said to have undergone a cycle, if it returns to its initial state at the end of the process. That is, for a cycle, the initial and final states are identical.

1.14 QUASI-STATIC (OR) QUASI–EQUILIBRIUM PROCESS

Briefly explain Quasi-Static process  (Nov/Dec 2013, Nov/Dec 2012 - AU)

When a process proceeds in such a manner that the system remains infinitesimally close to an equilibrium state at all times, it is called a quasi-static, (or) quasi-equilibrium process. A quasi-equilibrium process can be viewed as a sufficiently slow process that allows the system to adjust itself internally so that properties in one part of the system do not change any faster than those at other parts.

A quasi-static process is an idealized process and is not a true representation of an actual process. But many actual processes closely approximate it and they can be modeled as quasi-equilibrium with negligible error. Engineers are interested in quasi-equilibrium processes for two reasons as follows.

1. They are easy to analyze.
2. Work-producing devices (like engine) deliver maximum work when they operate on quasi-static processes.

Therefore, quasi-static processes serve as standards to which actual processes can be compared.

Consider a system of gas contained in a cylinder Fig 1.13(a). The system initially is in equilibrium state $P_1, V_1$ and $t_1$. The weight on the piston
is balanced by the pressure of the gas. If the weight is removed, there will be an unbalanced force between the system and surroundings, and under the gas pressure, the piston will move up till it hits the stops. The system comes to an equilibrium state - $P_2, V_2$ and $t_2$. But the inter-mediate states are non-equilibrium states and we cannot define its properties. So these two states (1 and 2) are joined by dotted line.

Now instead of single weight, we can place many number of small weights on the piston Fig 1.13(b). If we remove weights one by one very slowly from the piston, then the piston will move upward very slowly. The change of state of system from equilibrium state is infinitesimally small. So every state traced by the system is equilibrium state. The process, joined by all the equilibrium states passed by the system is called quasi-static process. ['Quasi’ means ‘almost’].
Quasi-static process is a reversible process.
A process which can be reversed in direction and the system retraces the same equilibrium states is known as reversible process.
Infinite slowness is the characteristic feature of a quasi-static process.

1.15 PATH FUNCTION AND POINT FUNCTION

Briefly explain the following: Point and path function (Nov/Dec 2013)

Refer the Fig 1.14. In this fig, there are many quasi-static paths such as A, B (or) C from state 1 to state 2. Since area under each curve represents the work for each process, the amount of work involved in path A, B and C are not equal. The work is not a function of the end states of the process but it depends on the path the system follows from state 1 to state 2. Hence, work is called a path function and \(dW\) is inexact (or) imperfect differential.

Thermodynamic properties like pressure, volume and temperature are point functions, because, for a given state, there is a definite value for each property. The change of property of a system is independent of the path the system follows during the change of state and it depends only on the initial and final states of the system. Hence \(dV\) (or) \(dP\) is an exact (or) perfect differentials.

For example, the integration of property is

\[
\int_{V_1}^{V_2} dV = V_2 - V_1
\]

i.e., Change in volume depends only on the end states of the system irrespective of the path the system follows.

But, work done in a quasi-static process between two given state depends on the path followed. And the integration of work transfer is

\[
\int_{1}^{2} dW \neq W_2 - W_1 \quad \text{Rather,} \quad \int_{1}^{2} dW = W_2 - W_1
\]
1.16 ENERGY

Energy is the ability of the physical system to perform work. Energy exists in several forms such as heat energy, kinetic energy, potential energy, light energy, electrical energy, thermal energy and so on. However, these energy transformations are constrained by a fundamental principle known as the “Conservation of Energy”, Principle. One way to state this principle is

“Energy can neither be created nor destroyed, but may transform from one form into another form”.

Another approach is that,

“the total energy of the isolated system remains constant”.

In S.I units, energy is represented as Joules (J)

\[ 1 \text{ Joule} = 1 \text{ Nm} = \frac{kg\ m}{s^2} \cdot m = \frac{kg\ m^2}{s^2} \]

Some forms of energy are explained as follows.

1.16.1 Different forms of energy


1. Potential Energy: (P.E)

It is the energy possessed by any system (or body) by virtue of its height (by virtue of its position above the ground level).

\[ P.E = mg\ Z \text{ (or) } mgh \text{ in J} \]

where \( Z = \text{ht above ground level.} \)

\[ P.E = gZ \text{ in J/kg} \]

2. Kinetic Energy: (K.E)

It is the energy possessed by any system (or body) by virtue of its velocity.

\[ K.E = \frac{1}{2} m\ v^2 \text{ in J} = \frac{1}{2} mC^2 \text{ in J} \]

where \( v = \text{Velocity in m/sec.} \)

\[ K.E = \frac{v^2}{2} \text{ in J/kg} = \frac{C^2}{2} \text{ in J/Kg} \]

To differentiate from volume \( V \), Velocity ‘V’ can be denoted by ‘C’ also.
3. **Pressure Energy**

   It is the energy possessed by any system (or body) by virtue of its pressure.

   \[ P_v = \frac{N}{m^2} \times \frac{m^3}{kg} = \frac{N \cdot m}{kg} = J/kg \]

   Sp. pressure energy = \( P_v \) in J/kg

4. **Internal Energy: \((u)\) in J**

   **Define internal energy.** (Nov/Dec 2011 - AU)

   It is the energy possessed by any system (or body) due to its molecular arrangement and motion of molecules. ‘\(u\)’ is the specific internal energy in J/kg.

---

**WORK AND HEAT TRANSFER**

1.17 **WORK TRANSFER**

In mechanics

\[ \text{Displacement} \]

Work = Force \times \text{distance moved} = F \times x

In thermodynamics, work is said to be done by a system, if sole effect on thing’s external to system can be reduced to the raising of weight.

The weight need not be raised. But the net effect can be utilized to raise the weight. Refer Fig 1.15.

Let \( A \) = Cross sectional area of the piston in \( m^2 \)

\( P \) = Pressure of gas at any time.

\( dl \) = Distance moved by the piston under gas pressure.

Work done = Force \times \text{distance travelled}

\[ = (PA) \times dl \]

\[ = P(Adl) = Pdv \]

Consider unit mass.

Then \( W = Pdv \) [where \( v \) = specific volume]

When the reversible process (Quasi-static process) takes place between states 1 and 2, then
Work done = \int_{1}^{2} Pdv

Therefore the work done by the gas during any reversible process is given by the area under the \( Pv \) diagram. (Shaded area).

### 1.17.1 Sign Convention

When the work is done by the system, it is positive work Fig 1.16(a).

When the work is done on the system, it is negative work Fig 1.16(b).

![Sign convention](image)

### 1.17.2 Work Transfer - A Path Function

State the thermodynamic definition of work. \( \text{(Nov/Dec 2015 - AU)} \)

Let us consider a system shown in Fig 1.17.

Work is also a form of energy transferred like heat and, therefore has energy unit kJ. The work done during a process between states 1 and 2 is denoted by \( W_{1-2} \) (or) simply \( W \). The work done per unit time is called power and denoted by \( P \). The unit of power is kJ/s (or) kW.

The system moves from a equilibrium state 1 to a final equilibrium state 2 through two different paths \( 1 - A - 2 \) and \( 1 - B - 2 \). The processes are quasi-static.

![Path function](image)
Since area under each curve represents the work for each process. \( 1 - A - 2 \) gives one value of work and \( 1 - B - 2 \) gives another value of work. So the work does not depend on the end state of the process. But it depends on the path the system it follows. So the work is called path function. Hence \( dW \) is an inexact or imperfect differential.

\[
W_2 - W_1 = \int_1^2 dW = \int_1^2 P\,dv
\]

**Note:**

Thermodynamic properties are point functions, since they depend only on the end states.

Example: Change in volume = \( V_2 - V_1 \)

Change in pressure = \( P_2 - P_1 \); Change in temperature = \( t_2 - t_1 \)

So the properties of the system depend on only end states and independent of the path the system follows. So the differentiation of point functions are exact or perfect differentials.

For a cyclic process, the initial and final states of the system are the same, and hence, the change in any property is zero.

\[
\oint dV = 0, \oint dP = 0, \oint dT = 0
\]

where the sign \( \oint \) denotes the cyclic integral for the closed path. Therefore, cyclic integral of a property is always zero.

**1.18 DIFFERENT MODES OF WORK**

**1.18.1 Electrical work**

When a current flows through a resistor (as shown in **Fig. 1.18**) taken as a system there is work transfer into the system. This is because the current can drive a motor, the motor can drive a pulley and the pulley can rise a weight.

The electrons crossing the system boundary do electrical work on the system. In an electrical field, electrons in a wire move under the effect of electromotive forces, doing work.

\[
W_e = VI = IR = V^2/R
\]
When current $I$ flows through a potential difference $V$, the electrical work done in the rate form is $W_e = VI$

where $W_e$ is the electrical power. In general, both $V$ and $I$ vary with time, and the electrical work done during a time interval $\Delta t$ is given by

$$W_e = \int_{1}^{2} VI \, dt \text{ (kJ)}$$

### 1.18.2 Shaft Work

Since the torque applied to the shaft is constant, the force $F$ applied is also constant (Fig 1.19).

Because Torque,

$$T = \text{Force} \times \text{radius} \ (r)$$

$$T = F \times r$$

For a specified constant torque $T$, the work done during $n$ revolutions is determined as follows.

We know $T = F \times r$ (or) $F = \frac{T}{r}$

This force acts through a distance $s$, which is related to the radius $r$ by

$$s = (2\pi r) \ n$$

Then the shaft work is determined from

$$W_{sh} = Fs = \left( \frac{T}{r} \right) (2\pi rn) = 2\pi n \ T \text{ kJ}$$

Also it can be given as $W_{sh} = \int_{1}^{2} T \, d\theta \text{ kJ}$ where $d\theta$ is the angular displacement of the shaft.

The power transmitted through the shaft is the shaft work done per unit time which can be given as

$$\dot{W}_{sh} = \frac{2\pi NT}{60 \times 1000} \text{ kW}$$

where $N =$ Number of revolutions per minute
### Power Transmission by the shaft of a car.

**Problem 1.1:** Determine the power transmitted through the shaft of a car when the torque applied is 300 N-m and the shaft rotates at a rate of 3500 revolutions per minute (rpm).

**Solution:**

We know \( \dot{W}_{sh} = \frac{2\pi NT}{60} \)

\[ \begin{align*}
\dot{W}_{sh} &= 2\pi \left( \frac{3500}{60} \right) \times \frac{300}{1000} \\
&= 109.96 \text{ kW}
\end{align*} \]

### 1.18.3 Spring Work

When a force is applied on a spring, the length of the spring changes. Refer Fig. 1.20 and Fig. 1.21. When the length of the spring changes by a differential amount \( dx \) under the influence of a force \( F \), the work done is

\[ \delta W_{spring} = F \, dx \]

To determine the total spring work, it is necessary to know the linear relationship between \( F \) and \( x \). For linear elastic springs, the displacement \( x \) is proportional to the force applied, i.e.

\[ F = kx, \quad \text{where } k = \text{spring constant in kN/m} \]

To get the total spring work

\[ \delta W_{spring} = F \, dx \Rightarrow F = kx \]

\[ \delta W_{spring} = kx \cdot dx \]
Integrating \[ \int_{1}^{2} \delta W_{spring} = \int_{1}^{2} kx \, dx \]

\[ W_{1-2} = \frac{1}{2} k (x_2^2 - x_1^2) \text{ kJ} \]

where \( x_1 \) and \( x_2 \) are the initial and the final displacements of the spring, respectively measured from the undisturbed position of the spring.

**Expansion of a gas against a spring.**

**Problem 1.2:** A piston-cylinder device contains 0.05 m\(^3\) of a gas initially at 200 MPa. At this state, a linear spring that has spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and compress the spring until the volume inside the cylinder doubles. If the cross sectional area of the piston is 0.25 m\(^2\), determine the fraction of work done by the gas against the spring to compress it.

**Solution**

**Assumptions:**

The expansion process is quasi-equilibrium

*The spring is linear in the range of interest.*

The enclosed volume at the final state is

\[ V_2 = 2V_1 = 2 \times 0.05 = 0.1 \text{ m}^3 \]

Then the displacement of the piston (and of the spring) becomes

\[ x = \frac{\Delta V}{A} = \frac{0.1 - 0.05}{0.25} = 0.2 \text{ m} \]

We know,

\[ W_{spring} = \frac{1}{2} k \left( x_2^2 - x_1^2 \right) \]

Here final displacement \( x_2 = 0.2m \)

Initial displacement \( x_1 = 0 \)

\[ W_{spring} = \frac{1}{2} \times 150 \times (0.2^2 - 0^2) = 3 \text{ kJ} \]
Problem 1.3: A piston-cylinder assembly contains air (ideal gas with \( \gamma = 1.4 \)) at 200 kPa and occupies a volume of 0.01 \( m^3 \). The piston is attached to one end of a spring and the other end of the spring is fixed to a wall. The force exerted by the spring on the piston is proportional to the decrease in the length of the spring from its natural length. The ambient atmospheric pressure is 100 kPa. Now, the air in the cylinder is heated till the volume is doubled and at this instant it is found that the pressure of the air in the cylinder is 500 kPa. Calculate the work done by the gas. (Nov/Dec 2015 - AU)

Solution: (a) Given

\[
\gamma = 1.4 \\
P_1 = 200 \text{ kPa} = 200 \text{ kN/m}^2 \\
V_1 = 0.01 \text{ m}^3 \\
V_2 = 2V_1 = 0.02 \text{ m}^3 \\
P_{atm} = 100 \text{ kPa} = 100 \text{ kN/m}^2 \\
P_2 = 500 \text{ kPa} = 500 \text{ kN/m}^2 \\
\therefore W = \frac{(P_1 + P_2)}{2} (V_2 - V_1) \\
\therefore W = \frac{(200 + 500)}{2} \times (0.02 - 0.01) \\
W = 3.5 \text{ kJ} \\
\]

\[
\therefore \text{Work done} = \text{Area of trapezium } A \ B \ C \ D = W = \frac{(P_1 + P_2)}{2} (V_2 - V_1) 
\]

1.18.4 Paddle wheel work (or) Stirring work

When the weight is lowered, and the paddle wheel turns, there is work transfer into the fluid system which gets stirred. Since the volume of the system remains constant,

\[
\int PdV = 0
\]

But work done \( dW = mg \, dz \), where \( m = \text{mass of weight lowered} \).

This work can be equated as, \( mg\, dz = Td\theta \)

To find total work transfer

\[
W = \int_{1}^{2} mg\, dz = \int_{1}^{0} Td\theta
\]
1.18.5 Flow Work

The flow work occurs only in flow process (or) open system. Unlike the closed systems, control volume involve mass flow across their boundaries, and some work is required to push the mass into (or) out of the control volume. This work is known as the flow work (or) flow energy, and is necessary for maintaining a continuous flow through a control volume.

Refer the Fig. 1.22 a and 1.22 b

Consider a fluid element of volume \( V \) as shown in fig. The fluid immediately upstream will force this fluid element to enter the control volume. Thus, it can be regarded as an imaginary piston.

When the fluid pressure is \( P \) and the cross sectional area of the fluid element is \( A \), the force applied on the fluid element by the imaginary piston is

\[
F = PA
\]

To push the entire fluid element into the control volume, the force must act through a distance \( L \). Thus, the work done in pushing the fluid element across the boundary (ie the flow work) is

\[
W_{\text{flow}} = FL = PAL = PV \quad (kJ)
\]

The flow work per unit mass is obtained by dividing both sides of this equation by the mass of the fluid element:

\[
W_{\text{flow}} = PV \quad (kJ/kg)
\]

The flow work relation is the same whether the fluid is pushed into (or) out of the control volume.
1.18.6 Workdone on Elastic solid bars

Solids are considered as linear springs because under the action of a force, they contract (or) elongate as shown in Fig. 1.23, and when the force is removed, they return to their original lengths, like a spring. This is true as long as the force is in the elastic range, that is, not large enough to cause permanent (plastic) deformations. Therefore, the equations given for a linear spring can also be used for elastic solid bars.

We can determine the work associated with the expansion (or) contraction of an elastic solid bar by replacing \( P \) by \( \sigma_n \).

Normal stress \( \sigma_n = \frac{F}{A} \)

Hence \( W_{\text{elastic}} = \int_1^2 \sigma_n dV = \int_1^2 \sigma_n A \, dx \) (kJ)

where \( A \) is the cross-sectional area of the bar. Note that the normal stress has pressure units.

1.18.7 Work Associated with the stretching of a Liquid Film

Refer the Fig. 1.24

Consider a liquid film such as soap film suspended on a wire frame. Some force is needed to stretch this film by the movable portion of the wire frame. This force is used to overcome the microscopic forces between molecules at the liquid-air interfaces.

These microscopic forces are perpendicular to any line in the surface and the force generated per unit length is called the surface tension \( \sigma_s \), where unit is N/m. Therefore, the work associated with the stretching of a film is also called surface tension work.
It is determined from, $W_{\text{surface}} = \int_{1}^{2} \sigma_s dA \ (\text{kJ})$

where $dA = 2b dx$ is the change in the surface area of the film. The factor 2 is due to the fact that the film has two surfaces in contact with air.

1.18.8 Work done per unit volume on a magnetic material

$$d \ W = H dI$$

$$W_{1-2} = \int_{1}^{2} H dI$$

where $H$ is the field strength and $I$ is the component of the magnetization field in the direction of the field.

The following equations summarize the different modes of work transfer.

Electrical work : $W_e = \int_{1}^{2} V I \ dt \ (\text{kJ})$

Shaft work : $W_{\text{shaft}} = \int_{1}^{2} T \ d\theta \ (\text{kJ})$

Spring work : $W_{\text{spring}} = \int_{1}^{2} k x \ dx \ (\text{kJ})$

Elastic solid bar : $W_{\text{elastic}} = \int_{1}^{2} \sigma_n A \ dx \ (\text{kJ})$

Surface Film : $W = \int_{1}^{2} \sigma_s dA \ (\text{kJ})$

Magnetic solid : $W = \int_{1}^{2} H dI$

Apart from the above, a very important work transfer is Displacement work for compressible fluid ie $\int P \ dv$ work. Which will be explained later in this book.
1.18.9 Free expansion work

Work transfer is a boundary phenomenon and can be identified only at the system boundaries. It is a form of energy in transit crossing the boundary. Let us, consider a gas separated from the vacuum by a partition as shown in fig. Now, let the partition be removed. The gas expands inside the chamber to fill the entire volume. This expansion of a gas against vacuum is called as free expansion.

Suppose, if we neglect the work involved with the removal of partition, and also consider gas and vacuum as a system, then there is no work transfer associated here, since no work crosses the boundary.

So,
\[ \int_{1}^{2} dw = 0 \]

But, if the gas alone is taken as a system as shown in Fig. 1.25 (b), then the gas is initially confined to a volume \( V_1 \), pressure \( P_1 \) and temperature \( T_1 \). And, after expansion, the gas takes Volume \( V_2 \), Pressure \( P_2 \) and temperature \( T_2 \).

Now, the work transfer can be calculated from the relation
\[ \int_{1}^{2} Pdv \neq 0 \]
Since the initial and final states are in equilibrium, the two end states can be located on the $P$–$V$ diagram as a dotted line.

But, if the vacuum space is divided into large number of small partitions and the partitions are removed one by one, then every state passed by the system is an equilibrium state and the work done can be estimated along with the $P$–$V$ diagram.

Besides these, however, in free expansion of a gas, there is no resistance to the fluid at the system boundary. Since vacuum does not offer any resistance, there is no work transfer involved in free expansion.

1.19 HEAT TRANSFER

Heat is defined as the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference.

The transfer of heat between two bodies in contact is called conduction. The transfer of heat between a wall and a fluid system in motion is called convection. The transfer of heat between two bodies separated by empty space (or) gases through electromagnetic waves is called radiation.

The temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer. Refer Fig 1.26.

1.19.1 Sign Convention

Normally, the heat is transferred from high temperature system to the low temperature system. The transfer of heat into a system is frequently referred to as heat addition and is taken as positive. The transfer of heat out of a system is called heat rejection and is taken as negative.

Heat is the energy which will be transferred from hot body to cold body between the system and surroundings. The heat transfer occurs because of temperature difference between system and surroundings Fig 1.27.

Heat flowing into the system is considered as positive. $(+ Q)$

Heat flowing out of the system is considered as negative. $(- Q)$
Refer Fig. 1.28. A process during which there is no heat transfer is called an **adiabatic process**. An adiabatic process should not be confused with an isothermal process. Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work.

![Fig. 1.27](adiabatic_process_diagram)

**Fig. 1.27**

During an adiabatic process, a system exchanges no heat with its surroundings.

![Fig. 1.28](adiabatic_system_diagram)

**Fig. 1.28**

As a form of energy, heat has energy units, kJ. The amount of heat transferred during the process between two states (states 1 and 2) is denoted by $Q_{1-2}$. Heat transfer per unit mass of a system is denoted by $q$.

$$ q = \frac{Q}{m} \text{ (kJ/kg)} $$

The rate of heat transfer i.e. the amount of heat transferred per unit time is denoted by $\dot{Q}$, where the **overdot** stands for the unit derivative (or) “per unit time”. The heat transfer rate $\dot{Q}$ has the unit kJ/s, which is equivalent to kW.
1.19.2 Heat Transfer – A Path Function

Define heat and show that heat is a path function and not a property.

(May/June 2012 - AU)

Heat transfer is also a path function. It depends on the path the system follows. It does not depend on the end states. So $\delta Q$ is inexact differential.

Refer Fig. 1.29. Area under $T$–$S$ diagram gives the heat transfer. In the processes $A$ and $B$, the end states are same, but the path are different. So the heat transfers in two processes $A$ and $B$ are different.

$$Q_{1 \to 2} = \int_{1}^{2} T ds$$

Note

$W_{1 \to 2} \neq W_1 - W_2$; $W_{1 \to 2} = $ Work transfer from 1 to 2.

$Q_{1 \to 2} \neq Q_1 - Q_2$; $Q_{1 \to 2} = $ Heat transfer from 1 to 2.

$s = $ entropy (which will be explained later).

$ds = $ change in entropy.

Path functions have Inexact differentials designated by the symbol $\delta$ (or $\delta t$). Therefore, a differential amount of heat (or) work is represented by $\delta Q$ or $\delta W$, respectively, instead of $dQ$ (or) $dW$.
1.19.3 Comparison of Heat and Work transfer

Enlist the similarities between work and heat. (Nov/Dec 2014 - AU)

<table>
<thead>
<tr>
<th>Work</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Work is a form of energy in transit</td>
<td>(i) Heat is also a form of energy in transit</td>
</tr>
<tr>
<td>(ii) Work is a path function</td>
<td>(ii) Heat is also a path function</td>
</tr>
<tr>
<td>(iii) Work is inexact differential</td>
<td>(iii) Heat is also inexact differential</td>
</tr>
<tr>
<td>(iv) All work can be converted into heat</td>
<td>(iv) But all heat can’t be converted into work</td>
</tr>
<tr>
<td>(v) Work is high grade energy</td>
<td>(v) Heat is low grade energy</td>
</tr>
</tbody>
</table>

1.20 HEAT CAPACITY

Heat capacity (also called as the thermal capacity is the ratio of the heat added to (or) heat removed from an object to the resulting temperature change.

It can also be defined as a product of mass and specific heat.

\[ C = mc = \text{kg} \times \frac{\text{J}}{\text{kgK}} = \frac{\text{J}}{\text{K}} \]

It is denoted by capital letter \( (C, C_p, C_v) \)

The S.I unit of heat capacity is Joule/Kelvin.

1.21 REVERSIBLE AND IRREVERSIBLE PROCESS

Reversible process

A process in which both system and its surroundings can be simultaneously returned to their initial states after the process has been completed.

Irreversible process

A process in which both system and its surroundings cannot be simultaneously returned to their initial states after the process has been completed.
Reversible process | Irreversible process
--- | ---
(i) Reversible process is an ideal process | Irreversible process is a natural process. (i.e.) all the processes occurring in nature are irreversible.
(ii) Reversible process attains equilibrium state at all the stages of the operation. | Irreversible process is in equilibrium only at the initial and final stages of operation.
(iii) It is a extremely slow process. | It occurs at measurable speed.
(iv) It takes infinite time for the process to occur. | It takes place in finite time.
(v) Workdone by a reversible process is greater than the corresponding irreversible process. | Workdone by an irreversible process is lesser than the corresponding reversible process.
(vi) It does not increase the entropy of the universe. | It increases the entropy of the universe (system & surroundings).

1.22 ZEROTH LAW OF THERMODYNAMICS

State the zeroth law of thermodynamics (Nov/Dec 2009 - AU)

If the systems A and B are in thermal equilibrium separately and A and C are in thermal equilibrium separately, then B and C should be in thermal equilibrium.

This law is the basis for temperature measurement.

1.22.1 Equality of temperature

When two bodies at different temperatures are brought into contact, after some time, they attain a common temperature and are then said to exist in thermal equilibrium.

The zeroth law was first formulated by R.H Fowler. This law explains the fundamental principles of thermodynamics, but was recognised after the formulation of first and second laws of thermodynamics. So it was named as ‘zeroth law’ since it should have preceded the first and second laws of thermodynamics.
1.23 CONCEPT OF TEMPERATURE AND THERMAL EQUILIBRIUM

The temperature of a body can be measured, by using a device known as thermometer. A common thermometer consists of a small amount of mercury in an evacuated glass tube. There are five different kinds of thermometer.

1. Constant volume gas thermometer
2. Constant pressure gas thermometer
3. Electrical resistance thermometer
4. Thermocouple
5. Mercury-in-glass thermometer

1.23.1 Thermometry

Thermometry is the science and practice of temperature measurement. Any measurable change in the thermometric probe (eg: extension of mercury in the tube of mercury-in-glass thermometer) can be used to mark temperature levels, that should later be calibrated as internationally agreed unit (like celsius, Fahrenheit, Kelvin, etc...)

Thermometry is broadly divided into two subfields.

(i) Contact thermometry (ii) Non-contact thermometry

1.23.2 Applications of thermometry

Temperature is one of the most measured physical parameters in science and technology, but thermometry is not only applied to measuring temperature, but also an indirect measure of many material properties like thermal capacities, relative humidity (by means of wet bulb (or) dew point temperatures) and enthalpy changes (by thermal analysis).

1.24 THERMOMETRIC PROPERTY - RELATIONSHIPS BETWEEN TEMPERATURE SCALES

Thermometric property is any physical property that changes measurably with temperature.

eg: Extension of mercury in mercury-in-glass thermometer

1.24.1 Reference points

A reference point is a ‘basis’ (or) a ‘standard’, for evaluation.

To calibrate the thermometer, first and foremost, we need to define a temperature scale. For defining the temperature scale, we have to choose reference points (reference temperature) also called as the fixed points.
For Example: In celsius scale, the fixed points are defined using the properties of pure water.

i.e., the lower fixed point or ice point, is the temperature of pure melting ice at normal atmospheric pressure.

Also, the higher fixed point or steam point, is the temperature of pure boiling water at normal atmospheric pressure.

Fixed point or reference points are classified into two categories.

(i) Two reference points method (or) two fixed points method (method used before 1954)

(ii) One reference point method (or) one fixed points method (method used after 1954)

(i) Two reference points method

In this method, two fixed points are used, i.e.,

(a) Ice point
(b) Steam point

(a) Ice point: Ice point is the temperature at which ice and water are in equilibrium at a pressure of 1 atm pressure.

(b) Steam point: Steam point is the temperature at which water vapour condenses at a pressure of 1 atm pressure.

Let $X$ be the thermometric property and $\theta (x)$ be the temperature of the system to be measured, then $\theta (x) = ax$ where $a$ is arbitrary constant.

The thermometer is first placed in contact with system whose temperature $\theta (X)$ to be measured & then in contact with one of the fixed point (steam or ice), whose temperature is $\theta (X_1)$, Thus

$$\frac{\theta (X)}{X} = \frac{\theta (X_1)}{X_1}$$

$$\begin{align*}
\frac{\theta (X_1) X_1}{\theta (X) X} &= X_1 \\
\theta (X) X &= \frac{X_1 X}{X_1} \\
\frac{\theta (X)}{X} &= \frac{X_2}{X} \\
\theta (X) &= \frac{X_2}{X} \\
\end{align*}$$

...(1)

Similarly, the thermometer at the temperature $\theta (s)$ is placed in contact with another fixed point of temperature $\theta (X_2)$. It gives

$$\frac{\theta (X_2)}{X} = \frac{X_2}{X}$$

...(2)
Subtract (2) from (1)

\[ \frac{\theta (X_1) - \theta (X_2)}{\theta (X)} = \frac{X_1 - X_2}{X} \]

\[ \theta (X) = \frac{\theta (X_1) - \theta (X_2)}{X_1 - X_2} \cdot X \]

The temperature interval, \( \theta (X_1) - \theta (X_2) \) between these two fixed points was chosen to be 100 degrees. The use of two fixed points was found unsatisfactory and later disregarded.

**(ii) One reference point method**

In this method, only one reference point is used, i.e., the triple point of water.

**Triple point of water:** It is a point at which three phases (solid, liquid and gas) i.e., ice, liquid water and water vapour exist in equilibrium with one another.

The temperature at which these states exist in equilibrium are 273.16 K or 273.16 degrees Kelvin.

The temperature of triple point of water \( \theta_t \) and thermometer property \( X_t \) is placed in contact with the body whose temperature \( \theta \) to be measured is

\[ \theta_t = a X_t \Rightarrow a = \frac{\theta_t}{X_t} = \frac{273.16}{X_t} \]

Therefore, \( \theta = a X \)

\[ \frac{\theta}{X} = \frac{\theta_t}{X_t} \Rightarrow \theta = \frac{273.16}{X_t} \cdot X \]

\[ \theta = 273.16 \times \frac{X}{X_t} \]

Therefore, the temperature of the triple point of water is called as the “standard fixed point of thermometry”.

---

**Fig: 1.30. Triple Point of Water**

- Pressure
- Ice (Solid) \( \text{At} 273.16 \text{ K (0.01°C)} \)
- Water (Liquid) \( \text{About 610 Pa} \)
- Water Vapor (Gas)
1.25 COMPARISON OF THERMOMETERS

<table>
<thead>
<tr>
<th>Thermometer</th>
<th>Thermometric property</th>
<th>Temperatures (using one fixed point method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Constant volume gas thermometer</td>
<td>Pressure $(P)$</td>
<td>$\theta (P) = 273.16 \frac{P}{P_t}$</td>
</tr>
<tr>
<td>2. Constant pressure gas thermometer</td>
<td>Volume $(V)$</td>
<td>$\theta (V) = 273.16 \frac{V}{V_t}$</td>
</tr>
<tr>
<td>3. Electric resistance thermometer</td>
<td>Resistance $(R)$</td>
<td>$\theta (R) = 273.16 \frac{R}{R_t}$</td>
</tr>
<tr>
<td>4. Thermocouple</td>
<td>Thermal e.m.f $(E)$</td>
<td>$\theta (E) = 273.16 \frac{E}{E_t}$</td>
</tr>
<tr>
<td>5. Liquid-in-glass thermometer</td>
<td>Length $(L)$</td>
<td>$\theta (L) = 273.16 \frac{L}{L_t}$</td>
</tr>
</tbody>
</table>

1.26 EQUATION OF STATE

The equation which relates the properties $P$, $v$, and $T$ is known as an **Equation of state**.

$\text{ie } f(P, v, T) = 0$

The simplest form of equation of state for the ideal gas is given below.

**Equation of state**: $P\bar{v} = RT$

[ $\bar{R}$ = universal gas constant in kJ/kg mol K and $\bar{v}$ = molar volume in m$^3$/kg mol ]

Also: $Pv = RT$

[ $R$ = characteristic gas constant in kJ/kg K]

$v$ = specific volume in m$^3$/kg

The above equation - Equation of state is also called **Characteristic gas equation**.

1.27 IDEAL GAS

An ideal gas is a theoretical gas consists of four gas variables i.e., Pressure $(P)$, Volume $(V)$, Temperature $(T)$ and number of mole of a gas $(n)$. Lastly, the constant in the equation is $\bar{R}$, which is the universal gas constant ($R = 8.314$ kJ/kg mol K)

$$P\bar{V} = \bar{R} \bar{T}$$

...$(1)$
\[ \bar{V} = \text{molar specific volume} \quad \frac{m^3}{\text{kg. mole}} \]

Divide the above equation by molecular weight \( \mu \), then the equation becomes,

\[ \frac{PV}{m} = RT; \quad R = \frac{\bar{R}}{\mu} \]

\( \mu = \text{molecular weight} \quad \left( \frac{\text{kg}}{\text{mol}} \right) \)

\( R = \text{characteristic gas constant} \quad (\text{kJ/kg k}) \)

Also, specific volume, \( v = \frac{\bar{V}}{\mu} \)

Also, \( m = n \mu \)

\( n = \text{number of mole of a gas} \quad (\text{kg moles}) \)

\( m = \text{mass of the gas} \quad (\text{kg}) \)

Therefore,

\[ PV = n \bar{R} T \]

\[ PV = n R \mu T \]

\[ PV = m RT \]

Equation 1,2,3 is called the ideal gas equation of state.

At very low pressure or density, all gases and vapour approaches ideal gas behaviour.

Practically, no ideal gas (perfect gas) exists in nature. However, hydrogen, oxygen, nitrogen and air behave as an ideal gas under normal condition.

Any equation that relates the pressure, temperature and specific volume of a substance is called an equation of state which is given here as

\[ P_V = R T \]

where the constant of proportionality \( R \) is called the gas constant. A gas that obeys this relation is called an ideal gas.
The ideal gas equation is very simple and thus very convenient to use. But, gases deviate from ideal-gas behaviour significantly. This deviation from ideal gas behaviour at a given temperature and pressure can accurately be accounted by the introduction of a correction factor called the compressibility factor $Z$ defined as

$$Z = \frac{PV}{RT} \quad \text{(or)} \quad PV = ZRT$$

It can also be expressed as

$$Z = \frac{V_{\text{actual}}}{V_{\text{ideal}}} \quad \text{where} \quad V_{\text{ideal}} = \frac{RT}{P}$$

Obviously, $Z = 1$ for ideal gases.

For real gases, $Z$ can be greater than (or) less than 1 when the gas deviates from ideal-gas behaviour.

### 1.28 NEW TEMPERATURE SCALES

#### 1.28.1 Ideal Gas Temperature Scale

A temperature scale which is nearly identical to the Kelvin scale is called ideal gas temperature scale.

Let us suppose a bulb of constant volume gas thermometer which is basically a rigid vessel filled with gas, usually hydrogen or helium at low pressure. Normally, at low pressures. “The temperature of a gas is proportional to its pressure at constant volume”. That is, the pressure of the gas of fixed volume varies linearly with temperature at sufficiently low pressures. Then the relationship between the temperature and the pressure of the gas in the vessel can be expressed as

$$T = a + bP$$

Where $a$ and $b$ are constants for gas thermometers and can be determined experimentally.

An ideal gas temperature scale can be developed by measuring the pressures of the gas in the vessel at two reference points i.e., ice and steam points - considering one straight line passing through fixed points, two equations can be generated which are sufficient to determine the constants $a$ & $b$. Now, the unknown temperature can be determined for the given corresponding pressure. The values of the constants will be different for different gas thermometer depending on the type and amount of gas in the vessel. But, if the values of ice and steam points are assigned as 0°C and
100°C, then the gas temperature scale is identical to the Celsius scale and the value of constant ‘a’ is determined as 
\(-273.15°C\) (corresponds to absolute zero pressure). This value of ‘a’ remains same for all gas thermometers regardless of the type and amount of gas in the vessel of the gas thermometer. That is, on a \(P-T\) diagram, all straight lines of different gas thermometers passing through the data points will intersect at \(-273.15°C\). This is the lowest temperature, that can be obtained by any gas thermometer. So we assign the value of ‘a’ as zero in case of absolute gas temperature scale. Then, the eqn reduces to \(T = bP\) and the temperature can be determined by specifying one fixed point.

It should be noted, that the absolute temperature scale is not a thermodynamic temperature scale, since it cannot measure very low and very high temperatures. It is used in the range in which the gas thermometers can be used and thus the thermodynamic temperature scale can be seen as an absolute temperature scale. (in gas thermometer range) that utilizes ideal gas which always acts as a low pressure gas regardless of the temperature. If such gas thermometer existed, it would read zero kelvin at absolute zero pressure, which corresponds to \(-273.15°C\) on Celsius scale.

The Kelvin scale is related to Celsius scale by

\[
T (K) = t (°C) + 273.15
\]

1.28.2 Celsius Temperature Scale

Celsius temperature scale is related to the Kelvin scale by

\[
t = T - 273.15°
\]

where \(t\) - Celsius temperature, \(T\) - Kelvin temperature

eg: At steam point, \(T = 373.15 K\) i.e., kelvin temperature of steam at 1 atm is 373.15 K

\[
t = 373.15 - 273.15
\]

\[
t = 100° C
\]
1.28.3 Contradiction in kelvin scale

The reference point of kelvin scale was changed to the triple point of water in 1954, with the value of 273.16K. The celsius scale was also redefined at the “Tenth General conference on weights and measures” in 1954 in terms of ideal gas temperature scale and single fixed point method, with the triple point of water was found to be 0.01°C.

But initially, the reference temperature chosen in the original kelvin scale was 273.15K (0°C), which is the temperature at which the water freezes and exists as solid-liquid mixture in equilibrium at standard atmospheric condition.

1.29 GAS THERMOMETERS

There are two main types of gas thermometers and they are
(i) constant volume gas thermometer
(ii) constant pressure gas thermometer

A simple form of constant-volume gas thermometer is shown in Fig. 1.32. The gas is enclosed in the bulb B and the pressure is recorded by the difference in the levels (h) of the mercury column. The mercury level at R is always adjusted so that it coincides with the mark. The pressure of the gas in the bulb is

\[ P = P_A + P_m \]

where \( P_A \) = Atmospheric pressure \( P_m \) = Manometric pressure

\[ P = P_A + P_m g h_m \]

\( P_m \) - density of mercury; \( h_m \) - height of mercury in liquid column

So, when the bulb is brought in contact with the system, whose temperature is to be measured, the bulb in course of time comes in thermal equilibrium with the system. The gas in the bulb expands, on being heated pushes the mercury downward. The flexible limb of the manometer is again adjusted after the difference in mercury level is recorded. Now the pressure of the gas can be evaluated.
Since the volume of the trapped gas is constant, then the ideal gas equation is,

$$\Delta T = \frac{V}{R} \Delta P$$

i.e., temperature is directly proportional to pressure.

Similarly, for constant pressure gas thermometer.

$$\Delta T = \frac{P}{R} \Delta V$$

temperature increases as a result of increase in volume.

1.30 THERMOCOUPLE

Thermocouple is a sensor, which is used to measure temperature. The working principle of thermocouple is based on seebeck effect.

Seebeck effect

In 1821, Thomas seebeck discovered that, if metals of two different materials were joined at both ends and if one end was at a different temperature than the other end, a voltage difference between two materials was produced. Thus, a current was created. This phenomenon is known as the seebeck effect and it is the basis for all thermocouples.

The joined ends of thermocouple are referred as hot junction and cold junction. The hot junction is the one where we measure the temperature & cold junction is the other where we connect the measurement instrument.
Due to the seebeck effect, a small voltage is created because of the temperature difference. This voltage is referred to as an EMF (electro motive force).

The voltage created by the thermocouple is extremely small & measured in terms of millivolt (one thousands of a volt).

Various combination of materials are used to makeup the thermocouple pairs. A thermocouple is normally calibrated by measuring the thermal emf at various known temperatures with the reference junction being kept at 0°C. The results of such measurements can usually be represented by a cubic equation of the form:

\[ \varepsilon = a + bt + ct^2 + dt^3 \]

where \( \varepsilon \) - thermal emf

\( a, b, c, d \) values are different for each thermocouple

**Advantages:**

Thermocouple comes in thermal equilibrium with the system rapidly, because of its lesser mass.

### 1.31 RESISTANCE THERMOMETERS

Resistance thermometers, also called as resistance temperature detectors (RTDs), are sensors used to measure temperature by correlating the resistance of the thermometer element with the temperature. Most commonly used resistance thermometers are called platinum resistance thermometer (PRT).

PRT measures temperature by a piece of pure platinum wire. The piece of platinum wire is referred to as a temperature sensor. This thermometer measures temperature to a high degree of accuracy and sensitivity, which makes it suitable as a standard for the calibration of other thermometers.

For smaller resistance, the following quadratic equation can be used

\[ R = R_o \left(1 + At + Bt^2\right) \]

where

- \( R_o \) = Resistance of the platinum wire at 0°C (when it is surrounding by melting ice)
- \( A, B \) = constants
- \( t \) = unknown temperature; \( R \) = known resistance
Solved Problems

Problem 1.4: The emf of the thermocouple with the hot junction \( t \)°C on gas thermometer scale and cold junction at ice point is given by

\[
 e = 0.256 - 4 \times 10^{-4} t^2 \text{ mV}
\]

The millivoltmeter is calibrated at ice and steam points. What this thermometer read in a plate where the gas thermometer reads 50°C

At ice point, \( t = 0 \)°C, \( e = 0 \)

At steam point

\( t = 100 \)°C \( e = 0.25 \times 100 - 4 \times 10^{-4} (100)^2 = 21 \text{ mV} \)

\( 50 \)°C, \( e = 0.25 \times 50 - 4 \times 10^{-4} (50)^2 = 12.5 - 1 = 11.5 \text{ mV} \)

So, when the gas thermometer reads 50°C, the thermocouple will read,

\[
 e = \frac{100}{21} \times 11.5 = 54.76 \text{°C}
\]

Problem 1.5: A platinum resistance thermometer has a resistance of 2.5 ohm at 0°C and 3.5 ohm at 100°C. Calculate the temperature when the resistance indicated is 5.5 ohm.

Given:

\[
 R \text{ at } 0 \text{°C} = 2.5 \text{ ohm}; R \text{ at } 100 \text{°C} = 3.5 \text{ ohm} \]

Let \( R = R_o (1 + \alpha t) \), \( R_o = 2.5 \) ohm

\[
 R_{100} = R_o (1 + \alpha t) \Rightarrow 3.5 = 2.5 (1 + \alpha 100)
\]

\[
 \frac{3.5}{2.5} - 1 = 100 \alpha \Rightarrow 0.4 = 100 \alpha \alpha = 0.4 \times 10^{-2}
\]

When \( R = 5.5 \) ohm

\[
 5.5 = 2.5 (1 + (0.4 \times 10^{-2}) t)
\]

\[
 \frac{5.5}{2.5} - 1 = 0.4 \times 10^{-2} t \Rightarrow 1.2 = 0.4 \times 10^{-2} t
\]

\[
 t = 300 \text{°C}
\]

Platinum resistance thermometer shows 5.5 ohm at 300°C
**Problem 1.6:** The temperature \( t \) on a thermometric scale is defined in terms of a property \( k \) by the relation

\[
t = a \ln k + b
\]

where \( a \) and \( b \) are constants.

The value of \( k \) are found to be 1.83 and 6.78 at the point (0°C) and steam point (100°C). Determine the temperature corresponding to a reading of \( k \) equal to 2.42 on thermometer.

**Given:**

\( k = 1.83 \) when \( t = 0 \); \( k = 6.78 \) when \( t = 100 \) \( k = 2.42 \) when \( t = ? \) \( t = a \ln k + b \)

(i) \( 0 = a \ln (1.83) + b \)

\[
0.6043 a + b = 0 \quad \text{...(1)}
\]

(ii) \( 100 = a \ln (6.78) + b \)

\[
1.914 a + b = 100 \quad \text{...(2)}
\]

On solving, \( a = 76.35 \quad b = -46.14 \)

(iii) \( t = 76.35 \ln (2.42) - 46.14 \)

\[
(76.35 \times 0.8838) - 46.14 = t
\]

\( t = 21.335 \degree C \)

**Problem 1.7:** A New scale \( N \) of temperature is divided in such a way that the freezing point of ice is 100\(^\circ\)N and the boiling point 400\(^\circ\)N what is the temperature reading on the new scale when the temperature in the celsius scale is 150°C. At what temp both the celsius and new temperature scale would be same.

**Given:**

<table>
<thead>
<tr>
<th>New temperature scale</th>
<th>Celsius scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice point = 100(^\circ)N</td>
<td>0(^\circ)C</td>
</tr>
<tr>
<td>Boiling point = 400(^\circ)N</td>
<td>100(^\circ)C</td>
</tr>
<tr>
<td>temperature = ?</td>
<td>when 150(^\circ)C</td>
</tr>
</tbody>
</table>
Let us take celsius scale on the \( x \)-axis and new temperature scale on the \( y \)-axis.

Slope, \( m = \frac{\Delta y}{\Delta x} = \frac{\Delta N}{\Delta C} \)

\[
= \frac{400 - 100}{100 - 0} = \frac{300}{100} = 3
\]

Slope-intercept form equation, is

\[
y = mx + b \quad \text{(y - intercept)}
\]

\[N = mc + b \quad \text{when} \quad x = 0 \quad \text{(N = 100 when} \quad C = 0)\]

\[
N = 3C + 100
\]

Temperature on the new scale, when temp or celsius scale is 150°C

\[N = 3 \times 150 + 100 = 550^\circ N\]

Also, the temp at which both scale readings are same.

At particular temperature, \( N = C \)

From the relation \( N = 3C + 100 \)

\[C = 3C + 100 \]

\[C - 3C = 100 \Rightarrow -2C = 100 \]

\[C = -50^\circ C \; \text{(or)} \; N = -50^\circ N\]

So, at \(-50^\circ\) both the scale would read the same value.
1.32 FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics, also known as the conservation of energy principle, provides a basis for studying the relationships among the various forms of energy and energy interactions. The first law of thermodynamics states that energy can be neither created nor destroyed; it can only change forms.

A rock at some elevation possesses some potential energy, and part of this potential energy is converted into kinetic energy as the rock falls. Experimental data show that the decrease in potential energy \( mg \Delta Z \) exactly equals to the increase in kinetic energy \( \frac{m(C_2^2 - C_1^2)}{2} \) when the air resistance is negligible; \( C_1 \) and \( C_2 \) are velocity at inlet and outlet respectively.

The increase in the energy of a potato in an oven is equal to the amount of heat transferred to it.

According to First Law of Thermodynamics

‘Heat can be converted into work and work can be converted into heat.’

For a closed system, Net heat transfer = Net work transfer.

\[ \oint Q = \oint W \]  
First law of Thermodynamics for closed system.

\( \oint \) is the cyclic integral for closed path. (i,e for cycle).
1.33 JOULES EXPERIMENT

Consider a closed system as shown in Fig 1.35. It consists of a known mass of water, insulated container having thermometer and a paddle wheel.

Now a certain amount of work is done on the system by means of a paddle wheel. The work can be measured as

\[ W_{1-2} = \text{Weight} \times \text{distance travelled} \]

The system was initially at temperature \( t_1 \) and after work transfer, the system temperature rises to \( t_2 \). The pressure is one atmospheric pressure. The process 1-2 is shown in Fig 1.36.

Now the insulation is removed. The heat transfer takes place between system and surroundings till the system returns to its original temperature \( t_1 \). The heat can be measured as

\[ Q_{2-1} = mC_p(\Delta t) \]
The system executes a cycle $1 \rightarrow 2 \rightarrow 1$.

Now we can see $W_{1 \rightarrow 2}$ is equal to $Q_{2 \rightarrow 1}$. It has been found that this work $W_{1 \rightarrow 2}$ is always proportional to the heat $Q_{2 \rightarrow 1}$ and the constant of proportionality is called the Joule’s equivalent (or) the mechanical equivalent of heat.

In this example, only two energy transfer occurs. If the cycle has so many heat and work transfers, we can get the same result as

$$\sum W_{\text{cycle}} = J(\sum Q)_{\text{cycle}}$$

where $J =$ Joule’s equivalent $= 1 \text{ Nm/J}$

Since it is unity, we can write this

$$\sum W_{\text{cycle}} = (\sum Q)_{\text{cycle}}$$

i.e. $\int dW = \int dQ$

### 1.34 PERPETUAL MOTION OF MACHINE OF FIRST KIND

**PMM 1**

PMM 1 is an imaginary engine which develops work without any heat input.

This machine is impossible.

This PMM 1 violates the I law of thermodynamics, since I law states that $\sum Q = \sum W$ in a cycle.

### 1.35 FIRST LAW APPLIED TO CLOSED SYSTEM UNDERGOING CHANGE OF STATE

*State the First law of thermodynamics for a non-flow process and for a cycle.*

*(May/June 2012 - AU)*

#### 1.35.1 Internal Energy

$$(\sum W)_{\text{cycle}} = (\sum Q)_{\text{cycle}}$$ applies only to systems undergoing cycles and the algebraic sum of all energy transfer across system boundaries is zero. But if a system undergoes a change of state during which both heat transfer and
work transfer are involved, the net energy transfer will be stored (or) accumulated within the system.

If \( Q \) = heat transfer and 
\( W \) = work transfer, then the net energy transfer \( (Q - W) \) will be stored in the system. Energy in storage is neither heat nor work and is given the name **Internal Energy** (or) simply energy of the system.

Therefore \( Q - W = \Delta E \)

where \( \Delta E \) is the increase in the energy of the system.

(or) \( Q = \Delta E + W \)

The change in energy of the system during a process is the sum of the changes in the internal, kinetic and potential energies and can be expressed as

\[
\Delta E = \Delta U + \Delta K E + \Delta P E
\]

where \( \Delta U = m (u_2 - u_1); \Delta K.E = \frac{1}{2} m (V_2^2 - V_1^2); \Delta P E = mg (Z_2 - Z_1) \)

When the initial and final states are specified, the values of the specific internal energies \( u_1 \) and \( u_2 \) can be determined directly from the property tables (or) thermodynamic property relations.

According to the first law of thermodynamics, when the system undergoes a process, both heat transfer and work transfer takes place. The net energy transfer is stored within the system and is known as stored energy (or) total energy of the system.

\[
Q - W = \Delta E \quad \text{First law of thermodynamics for process.}
\]

\[
\Delta E = \Delta (P.E + K.E + U)
\]

When \( P.E = 0; K.E = 0 \), then \( \Delta E = \Delta U \)

Then \( Q - W = \Delta U \). This is I law for process when \( P.E = 0 \) & \( K.E = 0 \). In other words, \( Q = W + \Delta U \).

### 1.35.2 Energy - A property of the system

Consider a system which changes its state from state 1 to state 2 by following the path A, and returns from state 2 to state 1 by following the path B. (Refer the following Fig 1.38). So the system undergoes a cycle. writing the first law for path A
\[ Q_A = \Delta E_A + W_A \]  
\[ Q_B = \Delta E_B + W_B \]

and for the path \( B \)

The processes \( A \) and \( B \) together constitute a cycle, for which

\[ (\Sigma W)_{\text{cycle}} = (\Sigma Q)_{\text{cycle}} \]

\[ (\text{or}) \ W_A + W_B = Q_A + Q_B \]

\[ (\text{or}) \ Q_A - W_A = W_B - Q_B \]

... (iii)

From equations (i), (ii) and (iii), it yields

\[ \Delta E_A = -\Delta E_B \]  
... (iv)

Similarly, if the system returns from state 2 to state 1 by following the path \( C \) instead of \( B \),

\[ \Delta E_A = -\Delta E_C \]  
... (iv)

From equations (iv) and (v)

\[ \Delta E_B = \Delta E_C \]  
... (v)

Therefore, it is proved that change in energy between two states of a system is the same, whatever path the system may follow in undergoing that change of state. Therefore, energy has a definite value for every state of the system. Hence, it is a point function and a property of the system.

SOLVED PROBLEMS

**Problem 1.8:** During a cycle, the sum of all heat transfers is \(-200\) kJ. The system completes 2 cycles per sec. Complete the following table, showing method for each item and compute the net rate of work output in kW.

<table>
<thead>
<tr>
<th>Process</th>
<th>( Q ) in kW</th>
<th>( W ) in kW</th>
<th>( \Delta E ) in kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a-b )</td>
<td>0</td>
<td>36</td>
<td>–</td>
</tr>
<tr>
<td>( b-c )</td>
<td>350</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>( c-d )</td>
<td>(-35)</td>
<td>–</td>
<td>(-610)</td>
</tr>
<tr>
<td>( d-a )</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

**Solution**

Process \( a-b, Q-W = \Delta E \)  
I.e.  
\[ Q = \Delta E + W \]  

(I law of thermodynamics for process)
0 = \Delta E + 36

So, \Delta E_{ab} = -36 \text{ kW}

Process \( b - c \), \( Q = \Delta E + W \)

\[
350 = \Delta E + 0 \Rightarrow \Delta E_{b-c} = 350 \text{ kW}
\]

Process \( c - d \), \( Q = \Delta E + W \)

\[
-35 = -610 + W \Rightarrow W_{c-d} = +575 \text{ kW}
\]

Process \( d - a \), \( (\Sigma Q)_{\text{cycle}} = -200 \text{ kJ} \)

The system completes 2 cycles/sec, so

\[
\Sigma Q = -200 \times 2 = -400 \text{ kW}
\]

\[
\Sigma Q = -400 \text{ kW} = Q_{a-b} + Q_{b-c} + Q_{c-d} + Q_{d-a}
\]

\[
-400 = 0 + 350 - 35 + Q_{d-a}
\]

So, \( Q_{d-a} = -715 \text{ kW} \)

Also, \( \oint dE = 0 \), since cyclic integral of any property is zero.

i.e. \( (\Sigma \Delta E)_{\text{cycle}} = 0 \)

\[
\Delta E_{a-b} + \Delta E_{b-c} + \Delta E_{c-d} + \Delta E_{d-a} = 0
\]

\[
-36 + 350 - 610 + \Delta E_{d-a} = 0
\]

\( \Delta E_{d-a} = 296 \text{ kW} \)

In Process \( d - a \), \( Q = \Delta E + W \)

\[
-715 = 296 + W
\]

\( W_{d-a} = -1011 \text{ kW} \)

<table>
<thead>
<tr>
<th>Process</th>
<th>( Q )</th>
<th>( W )</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a - b )</td>
<td>0</td>
<td>36</td>
<td>-36</td>
</tr>
<tr>
<td>( b - c )</td>
<td>350</td>
<td>0</td>
<td>350</td>
</tr>
<tr>
<td>( c - d )</td>
<td>-35</td>
<td>575</td>
<td>-610</td>
</tr>
<tr>
<td>( d - a )</td>
<td>-715</td>
<td>-1011</td>
<td>296</td>
</tr>
<tr>
<td>Total</td>
<td>( \Sigma Q = -400 )</td>
<td>( \Sigma W = -400 )</td>
<td>( \Sigma \Delta E = 0 )</td>
</tr>
</tbody>
</table>
Problem 1.9: A fluid system, contained in a piston and cylinder machine, passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is $-340$ kJ. The system completes 200 cycles per min.

<table>
<thead>
<tr>
<th>Process</th>
<th>$Q$ (kJ/min)</th>
<th>$W$ (kJ/min)</th>
<th>$\Delta E$ (kJ/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 2</td>
<td>0</td>
<td>4340</td>
<td>$-$</td>
</tr>
<tr>
<td>2 - 3</td>
<td>42000</td>
<td>0</td>
<td>$-$</td>
</tr>
<tr>
<td>3 - 4</td>
<td>$-4200$</td>
<td>$-$</td>
<td>$-73200$</td>
</tr>
<tr>
<td>4 - 1</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Complete the above table showing the method for each item, and compute the net rate of work output in kW. (May/June 2012 - AU)

Process 1 - 2: By 1st law of thermodynamics:

$$ Q = W + \Delta E $$

$$ 0 = 4340 + \Delta E $$

$$ \Delta E = -4340 \text{ kJ/min} $$

Process 2 - 3:

$$ Q = W + \Delta E $$

$$ 42000 = 0 + \Delta E $$

$$ \Delta E = 42000 \text{ kJ/min} $$

Process 3 - 4:

$$ Q = W + \Delta E $$

$$ -4200 = W - 73200 $$

$$ W = 69000 \text{ kJ/min} $$

Process 4 - 1:

$$ \Sigma Q = -340 \text{ kJ} $$

No. of cycle/min = 200

The total heat transfer/min = $-340 \times 200$

$$ = -68000 \text{ kJ/min} $$
\[ \Sigma \text{ cycle } Q = Q_{1-2} + Q_{2-3} + Q_{3-4} + Q_{4-1} \]
\[- 68000 = 0 + 42000 - 4200 + Q_{4-1} \]
\[Q_{4-1} = -1,05,800 \text{ kJ/min} \]

WKT, in a cycle process,
\[ \Sigma Q = \Sigma W \]
\[- 68000 = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1} \]
\[= 4340 + 0 + 69000 + W_{4-1} \]
\[W_{4-1} = -1,41,340 \text{ kJ/min} \]

Change in internal energy, 4 \(-\) 1
\[ \Delta E_{4-1} = Q - W = -1,05,800 - (-1,41,340) \]
\[= -1,05,800 + 1,41,340 \]
\[\Delta E_{4-1} = 35,540 \text{ kJ/min} \]

\[ \therefore \text{ Net Rate of work output} \]
\[\Sigma W = -68,000 \text{ kJ/min} = \frac{-68,000}{60} = 1133.33 \text{ kW} \]

<table>
<thead>
<tr>
<th>Process</th>
<th>(Q) (kJ/min)</th>
<th>(W) (kJ/min)</th>
<th>(\Delta E) (kJ/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (-) 2</td>
<td>0</td>
<td>4340</td>
<td>-4340</td>
</tr>
<tr>
<td>2 (-) 3</td>
<td>42000</td>
<td>0</td>
<td>42000</td>
</tr>
<tr>
<td>3 (-) 4</td>
<td>-4200</td>
<td>69000</td>
<td>-73200</td>
</tr>
<tr>
<td>4 (-) 1</td>
<td>-1,05,800</td>
<td>-1,41,340</td>
<td>35,540</td>
</tr>
</tbody>
</table>

**Problem 1.10:** When a system is taken from state a to state b along path acb, 84 kJ of heat flow into the system and the system does 32 kJ of work.

1. How much will the heat that flows into the system along the path adb be, if the work done is 10.5 kJ?
2. When the system is returned from b to a along the curved path, the work done on the system is 21 kJ. Does system absorb or liberate heat? and how much of the heat is absorbed or liberated?
3. If \(U_a = 0\) and \(U_d = 42\) kJ, find the heat absorbed in the processes ad and db.
Solution: Given \( Q_{acb} = +84 \text{kJ} \) [+ sign heat flows into the system]
\( W_{acb} = +32 \text{kJ} \) [+ sign work is done by the system]
\[
Q_{acb} = (U_b - U_a) + W_{acb}
\]
\[
84 = U_b - U_a + 32
\]
\[
U_b - U_a = 52 \text{kJ}
\]

1. To Find \( Q_{adb} \)
\[
Q_{adb} = (U_b - U_a) + W_{adb}
\]
\[
= 52 + 10.5 = 62.5 \text{kJ}
\]

2. To Find \( Q_{ba} \) (along curved path)
\[
Q_{ba} = (U_a - U_b) + W_{ba}
\]
\[
W_{ba} = -21 \text{kJ}
\]
\[
Q_{ba} = -52 - 21 = -73 \text{kJ}
\]

So the heat is liberated (73 kJ) from the system to surroundings.

3. If \( U_a = 0 \) and \( U_d = 42 \text{kJ} \), Then \( Q_{ad} = ?; Q_{db} = ? \)
\[
W_{adb} = 10.5 \text{kJ} \Rightarrow W_{adb} = W_{ad} + W_{db} = 10.5 \text{kJ}
\]
Here \( W_{db} = 0 \) since it is constant volume process.
\[
\text{[} \because W = \int pdv; \text{During } db \text{ process, } dv = 0 \text{]}
\]
So, \( W_{ad} = 10.5 \text{kJ} \)

To Find \( Q_{ad} \)
\[
Q_{ad} = (U_d - U_a) + W_{ad} = (42 - 0) + 10.5 = 52.5 \text{kJ}
\]
Heat absorbed during \( ad \) process = 52 kJ

To Find \( Q_{db} \)
\[
Q_{db} = (U_b - U_d) + W_{db}
\]
\[
Q_{adb} = 62.5 = Q_{ad} + Q_{db} = 52.5 + Q_{db}
\]
So,
\[
Q_{db} = 62.5 - 52.5 = 10 \text{kJ}
\]
Heat absorbed during \( db \) process = 10 kJ
1.36 SPECIFIC HEAT CAPACITIES

1.36.1 Specific heat $C \ [C_p \ (or) \ C_v]\$

The specific heat of a substance is defined as the amount of heat required to raise a unit mass of the substance through a unit rise in temperature. The symbol $C$ will be used for specific heat.

$$\therefore \ C = \frac{Q}{m \cdot \Delta t} \ J/kg \ K$$

where $Q =$ amount of heat transfer $(J)$

$m =$ mass of the substance $(kg)$; $\Delta t =$ rise in temperature $\cdot$ $(K)$

For gases, if the process is at constant pressure, it is $C_p$, and if the process is at constant volume, if is $C_v$. The product of mass and specific heat $(mC)$ is called the heat capacity of the substance.

1.36.2 Latent heat

The latent heat is the amount of heat transfer required to cause a phase change in unit mass of a substance at a constant pressure and temperature.

The latent heat of fusion is the amount of heat transferred to melt unit mass of solid into liquid.

The latent heat of vapourization is the quantity of heat required to vaporize unit mass of liquid into vapour.

The latent heat of sublimation is the amount of heat transferred to convert unit mass of solid to vapour.

1.36.3 Specific heat at constant volume:

The specific heat of a substance at constant volume $C_v$ is defined as the rate of change of specific internal energy with respect to temperature when the volume is held constant, ie

$$C_v = \left( \frac{\partial u}{\partial T} \right)_v$$

For a constant volume process $(\Delta u)_v = \int_{T_1}^{T_2} C_v \cdot dT$

The first law may be written for a closed stationary system composed of a unit mass of a pure substance

$$Q = \Delta u + W$$

(or) $\oint Q = du + dW$
For a process in the absence of work other than $Pdv$ work

$$d\ W = Pdv$$

$$\therefore d\ Q = du + Pdv$$

when the volume is held constant

$$(Q)_v = (\Delta u)_v$$

$$\therefore (Q)_v = \int_{T_1}^{T_2} C_v dT$$

ie Heat transferred at constant volume increases the internal energy of the system.

If the specific heat of a substance is defined in terms of heat transfer, then

$$C_v = \left(\frac{\partial Q}{\partial T}\right)_v$$

Since $u, T$ and $v$ are properties, $C_v$ is a property of the system. The product $mC_v$ is called the heat capacity at constant volume (J/K).

### 1.36.4 Enthalpy

**Define enthalpy. How is related to internal energy? (Nov/Dec 2012 - AU)**

The total heat content is called enthalpy. (H). Specific enthalpy is denoted by ‘$h$’

It is the heat transfer during constant pressure process

The sum of specific internal energy ($u$) and flow work ($Pv$) is called specific enthalpy ($h$).

$$h = u + Pv \text{ in J/kg}$$

The specific enthalpy ($h$) of a substance, is defined as $h = u + Pv$. It is a property of a system and intensive property.

From first law $d\ Q = du + Pdv$

At constant pressure, $Pdv = d\ (Pv)$

$$\therefore (d\ Q)_p = du + d\ (Pv) = d\ (u + Pv) = dh$$

where $h = u + Pv$ is the specific enthalpy, a property of the system.
Using the definition of enthalpy, and the equation of state of an ideal gas, we have

\[ h = u + P_v \] and \[ P_v = RT \] then \[ h = u + RT \]

Since \( R \) is constant, and \( u = f(T) \), it follows that the enthalpy of an ideal gas is also a function of temperature only.

\[ h = f(T) \]

### 1.36.5 Specific heat at constant pressure:

The specific heat at constant pressure \( C_p \) is defined as the rate of change of enthalpy with respect to temperature when the pressure is held constant.

\[ C_p = \left( \frac{\partial h}{\partial T} \right)_p \]

For a constant pressure process,

\[ (\Delta h)_p = \int_{T_1}^{T_2} C_p \, dT \]

The first law for a closed stationary system of unit mass

\[ d \, Q = du + P \, dv \]

Again \( h = u + P_v \)

\[ dh = du + P \, dv + v \, dP = d \, Q + v \, dP \]

\[ d \, Q = dh - v \, dP \]

\[ \therefore (d \, Q)_p = dh \]

(or) \( (Q)_p = (\Delta h)_p \)

\[ (Q)_p = \int_{T_1}^{T_2} C_p \, dT \]

The heat capacity at constant pressure is equal to \( mC_p \) (J/K)
1.37 THERMODYNAMIC PROCESSES

Thermodynamic processes are broadly classified into two types i.e.,

1.37.1 Application of first law to closed system (Non-flow process)

The following non-flow processes are reversible.

1. Constant volume process (Isochoric process) \( (V = \text{constant}) \) \( (n = \infty) \)
2. Constant pressure process (Isobaric process) \( (P = \text{constant}) \) \( (n = 0) \)
3. Constant temperature process (or) Isothermal process
   \( (t = \text{constant} \ (\text{or}) \ P v = \text{constant}) \) \( (n = 1) \)
4. Isentropic process (or) Reversible adiabatic process
   \( (P v^n = \text{constant}) \) \( (n = \gamma) \)
5. Polytropic process \( (P v^n = \text{constant}) \) \( (n = n) \)
1.38 CONSTANT VOLUME PROCESS

During this heating or cooling process, the volume remains constant.

\[ W_{1 \rightarrow 2} = \int P \, dv \]

\[ dV = 0; \quad W_{1 \rightarrow 2} = 0 \]

\[ Q = \Delta u + W = \Delta u + 0 \]

\[ \Delta u = mC_v(T_2 - T_1); \quad \text{So} \quad Q = mC_v(T_2 - T_1) \]

For any process \( PV^n = \text{constant} \)

Here \( n = \infty \) (infinite)

So, \( PV^n = C; \quad V^n = C/P \)

\[ V = (C/p)^{1/\infty} = 1 \]

i.e. \( V = \text{constant} \)

Change in enthalpy \( \Delta H = mC_p(T_2 - T_1) \)

\[ \int_{1}^{2} dH = \int_{1}^{2} dU + \int_{1}^{2} d(PV) \]
\[ H_2 - H_1 = (U_2 - U_1) + (P_2V_2 - P_1V_1) = mC_v(T_2 - T_1) + mR(T_2 - T_1) = (C_v + R) m(T_2 - T_1) \]

\[ H_2 - H_1 = mC_p(T_2 - T_1) \quad [\because C_p - C_v = R] \]

**Change in entropy** \[ \Delta S = mC_v \ln \left( \frac{T_2}{T_1} \right) \]

or \[ \Delta S = mC_v \ln \left( \frac{P_2}{P_1} \right) \]

**Problem 1.11:** Air occupies a space of 0.3 m^3 at a pressure of 2 bar and a temperature of 80°C. It is heated at constant volume, until the pressure is 8 bar. Determine (1) temperature at the end of the process (2) mass of air (3) change in internal energy (4) change in enthalpy (5) change in entropy during the process.

**Solution**

\[ V_1 = 0.3 \text{ m}^3; \quad P_1 = 200 \text{ kPa}; \quad T_1 = 80 + 273 = 353 \text{ K}; \quad P_2 = 800 \text{ kPa} \]

**Constant volume process**

To Find \( T_2 \) Temp. at the end of the process

\[ \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \]

\[ \frac{P_1}{T_1} = \frac{P_2}{T_2} \quad [\because V_1 = V_2] \]

\[ T_2 = P_2 \times \frac{T_1}{P_1} = 800 \times \frac{353}{200} = 1412 \text{ K} \]

To Find \( m \) - mass of air

\[ P_1V_1 = mRT_1 \]

\[ m = \frac{P_1V_1}{RT_1} = \frac{200 \times 0.3}{0.287 \times 353} = 0.59224 \text{ kg} \]

[\because R = 0.287 \text{ kJ/kg K} \text{ for air}]  

To Find \( \Delta U \) change in internal energy

\[ \Delta U = mC_v(T_2 - T_1) = 0.59224 \times 0.718(1412 - 353) = 450.32 \text{ kJ} \]

[\because C_v = 0.718 \text{ kJ/kg K} \text{ for air}]
To Find $\Delta H$ change in enthalpy
\[ \Delta H = mC_p(T_2 - T_1) = 0.59224 \times 1.005(1412 - 353) = 630.32 \text{ kJ} \]

[\because C_p = 1.005 \text{ kJ/kg K for air}]

To Find $\Delta S$ change in entropy
\[ \Delta S = mC_v \ln \left( \frac{T_2}{T_1} \right) = 0.59224 \times 0.718 \ln \left( \frac{1412}{353} \right) = 0.5895 \text{ kJ/kg K} \]

Problem 1.12: A rigid tank contains air at 600 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank are 70°C and 400 kPa, respectively. Determine the boundary work done during this process.

Solution:

A sketch of the system and the P-V diagram of the process are shown in Fig.

The boundary work can be determined as
\[ W = \int_1^2 PdV = 0 \]

because $dV = 0$ ie a rigid tank has a constant volume and $dV = 0$ in the above $W$ equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant - volume process is always zero. This is also evident from the $PV$ diagram of the process (ie the area under the process curve is zero).
1.39 CONSTANT PRESSURE PROCESS (OR) ISOBARIC PROCESS

\[ P = C; \quad n = 0 \]

During this process, (Heating (or) Cooling process) the pressure remains constant.

\[ W_{1-2} = \int_1^2 P \, dV = P \int_1^2 dV = P(V_2 - V_1) \]

Apply First law

\[ Q = \Delta U + W = mC_v(T_2 - T_1) + P(V_2 - V_1) \]

\[ = mC_v(T_2 - T_1) + mR(T_2 - T_1) = mC_p(T_2 - T_1) \quad [ \because \quad mRT = PV ] \]

\[ \therefore \quad C_p - C_v = R \text{ i.e } C_p = C_v + R \]

Also, \( Q = \Delta U + W = (U_2 - U_1) + P(V_2 - V_1) \)

\[ = (U_2 + P_2V_2) - (U_1 + P_1V_1) = H_2 - H_1 \]

So, \( Q = H_2 - H_1 = mC_p(T_2 - T_1) \)

Heat transfer = Change in enthalpy in constant pressure process.

Fig. 1.40 Constant Pressure Process (or) Isobaric Process
Also change in entropy $\Delta S = mC_p \ln \frac{T_2}{T_1}$.

(or) $\Delta S = mC_p \ln \left( \frac{V_2}{V_1} \right)$

For any process $PV^n = $ constant

In constant pressure process $n = 0$

So, $PV^0 = C \Rightarrow P = C$

**Problem 1.13:** A frictionless piston - cylinder device contains 5 kg of water vapour at 0.40 MPa and specific volume 0.5342 m$^3$/kg. Heat is now transferred to the system until the volume reaches 0.5951 m$^3$/kg at constant pressure. Determine the work done by the steam during this process.

**Solution:** A sketch of the system and P-V diagram of the process are shown here

Assumption: The expansion process is quasi-equilibrium

$W = \int_1^2 P \, dv = P (v_2 - v_1)$

$W = mP (v_2 - v_1)$

$= 5 \times (0.4 \times 10^3) (0.5951 - 0.5342) = 121.8 \text{ kJ}$

The positive sign indicates that the work is done by the system. That is, steam uses 121.8 kJ of its energy to do this work. The magnitude of this work could also be determined by calculating area under the process curve (ie rectangle) on the P-V diagram, which is simply $P \Delta V$ for this case.

**Problem 1.14:** 1 kg of gas with initial volume 2.5 m$^3$ is heated at constant pressure from 25°C to 220°C. Estimate the heat added, ideal work done, change in internal energy, change in enthalpy, change in entropy. Also calculate final pressure and final volume. Take $C_p = 0.984 \text{ kJ/kg K}$; $C_v = 0.728 \text{ kJ/kg K.}$
Solution: Given: \( V_1 = 2.5 \text{ m}^3; \ m = 1 \text{ kg}; \ T_1 = 25 + 273 = 298 \text{ K}; \)
\( T_2 = 220 + 273 = 493 \text{ K}; \ \ C_p = 0.984; \ C_v = 0.728 \)

Constant pressure process.

To Find \( R \) – Characteristic Gas Constant

For any gas \( C_p - C_v = R \) \( R = 0.984 - 0.728 = 0.256 \text{ kJ/kg K} \)

To Find \( W_{1-2} \)

\[
W_{1-2} = \int PdV = P(V_2 - V_1) = mR(T_2 - T_1)
\]
\[
= 1 \times 0.256(493 - 298)
\]
\[
= 49.92 \text{ kJ}
\]

To Find \( \Delta U \)

\[
\Delta U = mC_v(T_2 - T_1) = 1 \times 0.728(493 - 298)
\]
\[
= 141.96 \text{ kJ}
\]

To Find \( Q \)

Heat added \( \Delta Q = mC_p(T_2 - T_1) = 1 \times 0.984(493 - 298) \)
\[
= 191.88 \text{ kJ} = \Delta H
\]

[ \because \Delta Q = \Delta H \text{ for constant pressure process} ]

To Find Final Pressure \( P_2 \)

Since it is constant pressure process, \( P_1 = P_2 \)

\[
P_1V_1 = mRT_1
\]
\[
P_1 = \frac{1 \times 0.256 \times 298}{2.5} = 30.52 \text{ kPa} = P_2
\]

To Find Final Volume \( V_2 \)

\[
\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \ \text{since} \ P_1 = P_2
\]
\[
\frac{V_1}{T_1} = \frac{V_2}{T_2}, \ V_2 = \frac{V_1 \times T_2}{T_1}
\]
\[
V_2 = \frac{2.5}{298} \times 493 = 4.136 \text{ m}^3
\]
1.40 CONSTANT TEMPERATURE PROCESS (OR) ISOTHERMAL PROCESS \((PV = \text{CONSTANT}; \; T = \text{CONSTANT})\)

When a piston expands from high pressure to low pressure, the temperature will fall. But in this process, heat is added continuously to maintain the temperature constant. Similarly, during compression process, the pressure is raised. To avoid the temperature rise, the heat is liberated continuously. So the temperature will remain constant.

When the system temperature is maintained constant during a process, the process is called isothermal process or constant temperature process.

\[
\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}
\]

Since \(T_1 = T_2\),

\[P_1V_1 = P_2V_2\quad \text{i.e.}\quad PV = \text{constant}\]

(or) \(P = \frac{C}{V}\)
\[ W_{1 \rightarrow 2} = \int_{1}^{2} PdV = \int_{1}^{2} \frac{C}{V} dV = C \int_{1}^{2} \frac{dV}{V} = C \left[ \ln V \right]_{1}^{2} \]

\[ = PV \left[ \ln \frac{V_{2}}{V_{1}} \right] = mRT \left[ \ln \frac{V_{2}}{V_{1}} \right] \]

(or) \[ W_{1 \rightarrow 2} = PV \left[ \ln \frac{P_{1}}{P_{2}} \right] = mRT \left[ \ln \frac{P_{1}}{P_{2}} \right] \]

\[ Q = \Delta U + W \]
\[ Q = mC_v(T_2 - T_1) + W \]
\[ Q = 0 + W \]

So, \[ Q = W \]

\[ \Delta H = mC_p(T_2 - T_1) = 0 \]

\[ \Delta S = \text{Change in Entropy} \]
\[ \Delta S = mR \ln \left( \frac{V_{2}}{V_{1}} \right) \text{kJ/kg} \]

or \[ \Delta S = mR \ln \left( \frac{P_{1}}{P_{2}} \right) \text{kJ/kg K} \]

**Problem 1.15:** A piston - cylinder device initially contains 0.5 m$^3$ of air at 100 kPa and 80$^\circ$C. The air is now compressed to 0.1 m$^3$ in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

**Solution:** A sketch of the system and the P-V diagram of the process are shown in Fig.

**Assumptions:** The compression process is quasi-equilibrium. Air can be considered as an ideal gas.

For an ideal gas at constant temp.

\[ PV = mRT = C \text{ [constant]} \]

(or) \[ P = \frac{C}{V} \]
\[ W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{C}{V} \, dV \]

\[ = C \int_{V_1}^{V_2} \left( \frac{1}{V} \right) \, dV = C \left[ \ln V \right]_{V_1}^{V_2} = C \ln \frac{V_2}{V_1} \]

\[ W_{12} = P_1 V_1 \ln \frac{V_2}{V_1} \]

In the above equation, \( P_1 V_1 \) can be replaced by \( P_2 V_2 \) (or) \( mRT \). Also, \( \frac{V_2}{V_1} \) can be replaced by \( \frac{P_1}{P_2} \) because \( P_1 V_1 = P_2 V_2 \) ie

\[ \frac{V_2}{V_1} = \frac{P_1}{P_2} \]

Substituting numerical values in the above eqn.

\[ W_{1,2} = P_1 V_1 \ln \frac{V_2}{V_1} = 100 \times 0.5 \left[ \ln \frac{0.1}{0.5} \right] \]

\[ W_{1,2} = -80.472 \text{ kJ} \]

The negative sign indicates that this work is done on the system (a work input), which is always the case for compression process.

**Problem 1.16:** A quantity of air has a volume of 0.5 m³ at a pressure of 5 bar and 80°C. It is expanded in a cylinder at a constant temperature to a pressure of 1 bar. Determine the amount of work done and heat transfer during expansion.

**Solution:**

Given: \( V_1 = 0.5 \text{ m}^3; \ P_1 = 5 \text{ bar}; \ P_2 = 1 \text{ bar} \quad T_1 = 80 + 273 = 353 \text{ K}; \)

**Constant Temperature Process**

**To Find** \( m \)

\[ P_1 V_1 = mRT_1 \]
\[ m = \frac{P_1 V_1}{RT_1} \]
\[ = \frac{5 \times 10^2 \times 0.5}{0.287 \times 353} \]
\[ = 2.468 \text{ kg} \]

To Find \( V_2 \)

\[ P_1 V_1 = P_2 V_2 \]

\[ V_2 = \frac{P_1 V_1}{P_2} \]
\[ = \frac{5 \times 10^2 \times 0.5}{1 \times 10^2} \]
\[ = 2.5 \text{ m}^3 \]

To Find \( W \)

\[ W = mRT_1 \ln \frac{V_2}{V_1} \]
\[ = 2.468 \times 0.287 \times 353 \left( \ln \frac{2.5}{0.5} \right) \]
\[ = 402.42 \]

\( W_{1-2} = 402.42 \text{ kJ} \)

\( Q = W \text{ in Isothermal process} \)

So \( Q = W = 402.42 \text{ kJ} \)

Note: For any process \( PV^n = C \)

For isothermal process \( n = 1 \)

So, \( PV = \text{constant} \)
Problem 1.17: A mass of 1.5 kg of air is compressed in a quasi static process from 0.1 MPa to 0.7 MPa for which $PV = \text{Constant}$. The initial density of air is 1.16 kg/m$^3$. Find the work done by the piston to compress the air.

Solution:
Given data:

- $m = 1.5 \text{ kg}$; ($PV = C \therefore \text{ Isothermal process}$)
- $P_1 = 0.1 \text{ MPa} = 100 \text{ kN/m}^2$
- $P_2 = 0.7 \text{ MPa} = 700 \text{ kN/m}^2$
- Initial density ($\rho_1$) = 1.16 kg/m$^3$
- $v_1 = \frac{1}{1.16} = 0.8621 \text{ m}^3/\text{kg}$
- $V_1 = m \cdot v_1 = 1.5 \times 0.8621 = 1.2931 \text{ m}^3$

To find Work done $W$:

For isothermal process $P_1 V_1 = P_2 V_2$

$V_2 = \frac{P_1 V_1}{P_2}$

$= \frac{100 \times 1.2931}{700}$

$= 0.1847 \text{ m}^3$

Work done for isothermal process $W = P_1 V_1 \ln \left( \frac{V_2}{V_1} \right)$

$W = 100 \times 1.2931 \ln \left( \frac{0.1847}{1.2931} \right) = -251.6 \text{ kJ}$

(−ve sign indicates that the work is done on the system)
1.41 REVERSIBLE ADIABATIC PROCESS (OR) ISENTROPIC PROCESS

\[(PV)^k = C; \quad (S = \text{constant})\]

Reversible - No friction
Adiabatic - No heat transfer \((Q = 0)\)

If there is no friction and no heat transfer, then there is no entropy change.

So reversible adiabatic process is isentropic process.

\[Q = \Delta U + W\]

Fig. 1.43 Reversible adiabatic process

\[W_{1 \rightarrow 2} = \int P \, dV\]

Here \((PV)^k = C\)

\[\therefore P = \frac{C}{V^k}\]

\[W = \int_1^2 \frac{C}{V^k} \, dV\]
\[
C \int_{V_1}^{V_2} \frac{1}{V^\gamma} dV = P V^\gamma \int_{V_1}^{V_2} V^{-\gamma} dV = PV^\gamma \left[ \frac{V_2^{\gamma + 1} - V_1^{\gamma + 1}}{-\gamma + 1} \right]
\]

\[
P_2 V_2^{1-\gamma} - P_1 V_1^{1-\gamma} = \frac{P_2 V_2 - P_1 V_1}{1-\gamma}
\]

\[
W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{1-\gamma} = \frac{mR(T_2 - T_1)}{1-\gamma}
\]

Alternate Method:

\[
Q = \Delta U + W
\]

So, \( Q = 0 \) since it is adiabatic

\[
W = -\Delta U
\]

Also \( W = -\Delta U = -[mC_v(T_2 - T_1)] \)

Note: \( C_v = \frac{R}{\gamma - 1} \), So \( W = -\left[ \frac{mR}{\gamma - 1} (T_2 - T_1) \right] \)

\[
W = \frac{mR(T_2 - T_1)}{1-\gamma}
\]

Change in Enthalpy \( \Delta H = mC_p(T_2 - T_1) \)

Change in Entropy \( \Delta S = 0 \)

For any process \( PV^n = C \)

For this process \( PV^\gamma = C \)

So, \( n = \gamma \)

**Problem 1.18:** Air at 1.02 bar, 22°C initially occupies a cylinder volume of 0.015 m³ is compressed isentropically by a piston to a pressure of 6.8 bar. Determine, (i) the final temperature (2) the final volume (3) the work done.

**Solution**

\[
P_1 = 1.02 \text{ bar};
\]

\[
T_1 = 22 + 273 = 295 \text{ K}
\]

\[
V_1 = 0.015 \text{ m}^3; P_2 = 6.8 \text{ bar}
\]
Isentropic compression

1. To Find $T_2$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

[We can get this relation from $pV^\gamma = C$]

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$T_2 = 295 \left(\frac{6.8}{1.02}\right)^{0.4} = 507.3 \text{ K}$$

[\therefore \ \gamma = 1.4 \text{ for air}]

2. To Find Final Volume $V_2$

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$\left(\frac{V_2}{V_1}\right)^\gamma = \frac{P_1}{P_2}$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} = 0.015 \left(\frac{1.02}{6.8}\right)^{\frac{1}{1.4}} = 3.869 \times 10^{-3} \text{ m}^3$$

3. Work done

$$W = \frac{P_2V_2 - P_1V_1}{1 - \gamma}$$

$$= \frac{(6.8 \times 10^2)(3.869 \times 10^{-3}) - (1.02 \times 10^2 \times 0.015)}{1 - 1.4}$$

$$= -2.7523 \text{ kJ}$$
Problem 1.19: 10 kg of gas at 10 bar and 400° C expands reversibly and adiabatically to 1 bar. Find the work done and change in internal energy.

Solution:

Given data:

\[ m = 10 \text{ kg}; \ P_1 = 10 \text{ bar} = 1000 \text{ kN/m}^2; \]
\[ P_2 = 1 \text{ bar} = 100 \text{ kN/m}^2 \]
\[ T_1 = 400^\circ \text{ C} + 273 = 673 \text{ K} \]

For Isentropic (reversible adiabatic) process,

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \Rightarrow T_2 = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \times T_1 \]

\[ T_2 = \left( \frac{100}{1000} \right)^{\frac{1.4 - 1}{1.4}} \times 673 = 348.58 \text{ K} \]

To find Work done \( (W) \)

\[ W = \frac{mR(T_1 - T_2)}{\gamma - 1} \]

\[ W = \frac{10 \times 0.287 \times (673 - 348.58)}{1.4 - 1} = 2327.71 \text{ kJ} \]

\[ W = 2327.71 \text{ kJ} \]

To find Change in internal energy \( \Delta U \)

\[ \Delta U = mC_v(T_2 - T_1) \]

\[ = 10 \times 0.718 \times (348.58 - 673) = -2329.33 \text{ kJ} \]

\[ \Delta U = -2329.33 \text{ kJ} \]
Basic Concepts and First Law

\[ P_1 V_1^\gamma = P_2 V_2^\gamma \]

\[ \left( \frac{V_2}{V_1} \right)^\gamma = \frac{P_1}{P_2} \]

\[ \frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^\gamma \]

\[ \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \]

\[ \frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} \times \left( \frac{P_1}{P_2} \right)^\gamma \]

\[ = \frac{P_2}{P_1} \times \left( \frac{P_1}{P_2} \right)^\gamma \]

\[ = \left( \frac{P_2}{P_1} \right)^{1 - \frac{1}{\gamma}} \]

\[ = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \]

So,

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \]
POLYTROPIC PROCESS: \((PV^n = \text{constant})\) \((n = n)\)

Generally all the processes are polytropic process. When the index of expansion (or) compression 'n' is changed, the process name is changed.

When index \(n = \infty\), it is called constant volume process
when \(n = 0\), it is constant pressure process
when \(n = 1\), it is isothermal process
when \(n = \gamma\), it is isentropic process.

When \(n = n\), then it is polytropic process.
The \(n\) varies from 0 to \(\infty\).

During actual expansion and compression processes of gases, pressure and volume are often related by \(PV^n = C\), where \(n\) and \(C\) are constants. A process of this kind is called a **polytropic process**. Let us derive a general expression for the work done during a polytropic process.

We know, \(PV^n = C\) for polytropic process. \(p = \frac{C}{V^n} = CV^{-n}\)

Substitute the relation in \(W = \int_1^2 PdV\) equation

We get \(W = \int_1^2 PdV = \int_1^2 CV^{-n} \, dv\)

\[
W = C \left[ \frac{V^{n+1}}{1-n} \right]_1^2 = PV^n \left[ \frac{V_2^{1-n} - V_1^{1-n}}{1-n} \right] = \left[ \frac{P_2 V_2^{1-n} - P_1 V_1^{1-n}}{1-n} \right]
\]
\[ W = \frac{P_2 V_2 - P_1 V_1}{1 - n} \]

Note here that \( C = PV^n = P_1 V_1^n = P_2 V_2^n \)

For an ideal gas, \( PV = mRT \),

The work done equation can be written as
\[ W = \frac{mR(T_2 - T_1)}{1 - n} \]

This \( PV^n \) can be used for any process by substituting appropriate value for \( n \).

Polytropic Work done can be calculated by replacing \( \gamma \) in the isentropic process work done by \( n \).

1. \[ W = \frac{P_2 V_2 - P_1 V_1}{1 - n} = \frac{mR(T_2 - T_1)}{1 - n} \]

(or) \[ W = \frac{P_1 V_1 - P_2 V_2}{n - 1} = \frac{mR(T_1 - T_2)}{n - 1} \]

\( Q \neq 0 \)
\[ Q = \Delta U + W = mC_v(T_2 - T_1) + \frac{mR(T_2 - T_1)}{1 - n} \]

Note \( \frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{\frac{1}{\gamma}}; \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n}{n-1}} \)

2. \[ \Delta U = mC_v(T_2 - T_1) = \frac{mR(T_2 - T_1)}{\gamma - 1} \]
\[ Q = \Delta U + W = \left[ \cdot \cdot \cdot C_v = \frac{R}{\gamma - 1} \right] \]
3. So in polytropic process $Q = \left( \frac{\gamma - n}{\gamma - 1} \right) W$ (or)

$$Q = mc_n(T_2 - T_1) \text{ where } C_n = C_V \left( \frac{\gamma - n}{1 - n} \right)$$

4. Change in enthalpy $\Delta H = mc_p(T_2 - T_1)$

5. Change in entropy $\Delta S = \frac{\gamma - n}{\gamma - 1} mR \ln \left( \frac{V_2}{V_1} \right)$ (or) $\Delta S = mc_n \ln \left( \frac{T_2}{T_1} \right)$

By studying all the non-flow processes we can draw a tabular column to know $W, Q, \Delta U, \Delta H, \Delta S$ for ready made use.

By referring the tabular column in the nearby pages, we can solve any type of problem in non-flow process.

**Problem 1.20:** Air at 300°C and 10 bar expands to 3 bar reversibly following the law $PV^{1.35} = C$. Determine the work done per kg of air, heat transfer and change in entropy.

**Solution**

Given: $T_1 = 300 + 273 = 573$ K; $P_1 = 10$ bar = 1000 kPa

$P_2 = 300$ kPa; $PV^{1.35} = C$; mass = 1 kg
To Find W

\[ W = \frac{mR(T_1 - T_2)}{n - 1} \]

\[ T_2 = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} T_1 \]

\[ T_2 = \frac{573}{1000} \left( \frac{300}{1000} \right)^{\frac{0.35}{1.35}} = 419.4 \text{ K} \]

[Alternate Method: \( C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) \) = 0.718 \( \left( \frac{1.4 - 1.35}{1 - 1.35} \right) = -0.10257 \)

\[ Q = mC_n(T_2 - T_1) = 1 \times (-0.10257)(419 - 573) = 15.755 \text{ kJ} \]

\[ \Delta S = mC_p \ln \left( \frac{T_2}{T_1} \right) = 1 \times (-0.10257) \ln \left( \frac{419.4}{573} \right) \]

\[ \Delta S = 0.032 \text{ kJ/kg K} \]

Problem 1.21: Gas at 1.5 bar abs. and 20°C in a closed vessel is compressed to 10 bar abs. Its temperature then becomes 180°C. If the compression follows the law \( PV^\gamma = C \), find the value of ‘n’.

Solution:

Given data:

\[ P_1 = 1.5 \text{ bar} = 150 \text{ kN/m}^2 ; \quad T_1 = 20^\circ \text{C} = 20 + 273 = 293 \text{ K} \]

\[ P_2 = 10 \text{ bar} = 1000 \text{ kN/m}^2 ; \quad T_2 = 180^\circ \text{C} = 180 + 273 = 453 \text{ K} \]

Law \( PV^\gamma = \text{Constant} \). So polytropic process.

To find The value of the polytropic index ‘n’

From polytropic relation

\[ \left( \frac{T_2}{T_1} \right)^{\frac{n-1}{n}} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \]

Taking \( \ln \) on both sides

\[ W = \frac{1 \times 0.287(573 - 419.4)}{0.35} = 125.98 \text{ kJ} \]

\[ Q = \frac{\gamma - n}{\gamma - 1} W = \frac{1.4 - 1.35}{1.4 - 1} \times 125.98 = 15.747 \text{ kJ/kg} \]

Law \( PV^\gamma = \text{Constant} \). So polytropic process.
Problem 1.22: A cylinder contains 1 m$^3$ of gas at 100 kPa and 100$^\circ$ C, the gas is polytropically compressed to a volume of 0.25 m$^3$. The final pressure is 600 kPa. Determine (a) mass of the gas (b) the value of index ‘n’ for compression (c) the change in internal energy of the gas (d) the heat transfer by the gas during compression. Assume $R = 0.287$ kJ/kg K and $\gamma = 1.4$.

Solution: Given data:

$V_1 = 1$ m$^3$; $P_1 = 100$ kPa = 100 kN/m$^2$; $T_1 = 100^\circ$ C + 273 = 373 K

$V_2 = 0.25$ m$^3$; $P_2 = 600$ kPa = 600 kN/m$^2$; $R = 0.287$ kJ/kg K; $\gamma = 1.4$

To find $m$:

By equation of state for gas

$$P_1V_1 = mRT_1 \Rightarrow m = \frac{P_1V_1}{RT_1}$$

$$m = \frac{100 \times 1}{0.287 \times 373} = 0.934 \text{ kg}$$

To find $n$:

The value of index ‘n’ is given by

$$n = \frac{\ln \left( \frac{P_2}{P_1} \right)}{\ln \left( \frac{V_1}{V_2} \right)} = \frac{\ln \left( \frac{600}{100} \right)}{\ln \left( \frac{1}{0.25} \right)} = \frac{1.792}{1.386} = 1.2925$$

$n = 1.2925$
To find $\Delta U$

Change in internal energy, $\Delta U = mC_v(T_2 - T_1)$

To find $T_2$:

From polytropic equation

$$
\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}
$$

$$
T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \times T_1
$$

$$
T_2 = \left(\frac{600}{100}\right)^{\frac{1.2925 - 1}{1.2925}} \times 373 = 559.5 \text{ K}
$$

$$
T_2 = 559.5 \text{ K}
$$

$\Delta U = 1 \times 0.718 (559.5 - 373)$

$\Delta U = 134 \text{ kJ}$

To find Heat transfer $Q$

$$
Q = W \times \left(\frac{\gamma - n}{\gamma - 1}\right)
$$

Work done

$$
W = \frac{mR(T_1 - T_2)}{n - 1}
$$

$$
W = \frac{1 \times 0.287 \times (373 - 559.5)}{1.2925 - 1}
$$

$$
W = -182.99 \text{ KJ}
$$

$$
Q = -182.99 \times \left(\frac{1.4 - 1.2925}{1.4 - 1}\right)
$$

$$
Q = -49.2 \text{ kJ}
$$

Alternate Method

$$
C_n = C_v \left(\frac{\gamma - n}{1 - n}\right)
$$

$$
= 0.718 \left(\frac{1.4 - 1.2925}{1 - 1.2925}\right)
$$

$$
= -0.2639
$$

$$
Q = m \ C_n \ (T_2 - T_1)
$$

$$
Q = 1 \times (-0.2639) (559.5 - 373)
$$

$$
Q = -49.2 \text{ kJ}
$$
For all processes, \( \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \) and \( PV = mRT \)

<table>
<thead>
<tr>
<th>Process</th>
<th>( pV^n = C )</th>
<th>( W ) (Work done)</th>
<th>( \Delta U ) (Change in Internal energy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Constant volume process</td>
<td>( n = \infty ) ( V = C )</td>
<td>0 (zero)</td>
<td>( mC_v(T_2 - T_1) )</td>
</tr>
<tr>
<td>2. Constant pressure process</td>
<td>( n = 0 ) ( P = C )</td>
<td>( W = P(V_2 - V_1) ) ( ) (or) ( W = mR(T_2 - T_1) )</td>
<td>( mC_v(T_2 - T_1) )</td>
</tr>
<tr>
<td>3. Constant temperature process</td>
<td>( n = 1 ) ( PV = C ) ( T = C )</td>
<td>( PV \ln \left( \frac{V_2}{V_1} \right) ) ( ) or ( PV \ln \left( \frac{P_1}{P_2} \right) ) ( ) or ( mR \ln \left( \frac{V_2}{V_1} \right) )</td>
<td>0 (zero) (since ( T_1 = T_2 ))</td>
</tr>
<tr>
<td>4. Isentropic process (or)</td>
<td>( n = \gamma ) ( PV^n = C )</td>
<td>( \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} ) ( ) (or) ( \frac{mR(T_1 - T_2)}{\gamma - 1} )</td>
<td>( mC_v(T_2 - T_1) ) or ( \Delta U = -W )</td>
</tr>
<tr>
<td>5. Polytropic process</td>
<td>( n = n ); ( pV^n = C )</td>
<td>( \frac{P_1 V_1 - P_2 V_2}{n - 1} ) ( ) (or) ( \frac{mR(T_1 - T_2)}{n - 1} )</td>
<td>( mC_v(T_2 - T_1) )</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\text{ie } P_1V_1 &= mRT_1; \quad P_2V_2 = mRT_2 \ldots
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>( Q )</th>
<th>( \Delta H )</th>
<th>( \Delta S )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( mC_v(T_2 - T_1) )</td>
<td>( mC_v(T_2 - T_1) )</td>
<td>( mC_v \ln \left( \frac{T_2}{T_1} \right) ) (or) ( mC_v \ln \left( \frac{P_2}{P_1} \right) )</td>
<td>( PdV = 0 ) since ( dV = 0 ); ( Q = \Delta U ); ( V_1 = V_2 )</td>
</tr>
<tr>
<td>2.</td>
<td>( mC_p(T_2 - T_1) )</td>
<td>( mC_p(T_2 - T_1) )</td>
<td>( mC_p \ln \left( \frac{T_2}{T_1} \right) ) (or) ( mC_p \ln \left( \frac{V_2}{V_1} \right) )</td>
<td>( Q = \Delta H ) ( P_1 = P_2 )</td>
</tr>
<tr>
<td>3.</td>
<td>( Q = W ) ( = PV \ln \left( \frac{V_2}{V_1} \right) ) ( = PV \ln \left( \frac{P_1}{P_2} \right) )</td>
<td>( 0 ) (zero)</td>
<td>( mC_p \ln \left( \frac{V_2}{V_1} \right) ) (or) ( mC_p \ln \left( \frac{P_1}{P_2} \right) )</td>
<td>( Q = W ); ( P_1V_1 = P_2V_2 ); ( T_1 = T_2 )</td>
</tr>
<tr>
<td>4.</td>
<td>Zero ( (\text{No Heat Transfer}) )</td>
<td>( mC_p(T_2 - T_1) )</td>
<td>( 0 ) (zero) since ( s_1 = s_2 )</td>
<td>( mC_p \ln \left( \frac{T_2}{T_1} \right) ) (or) ( mC_p \ln \left( \frac{V_2}{V_1} \right) ) (or) ( mC_p \ln \left( \frac{P_1}{P_2} \right) )</td>
</tr>
<tr>
<td>5.</td>
<td>( \gamma - n \frac{\gamma}{\gamma - 1} \times W ) ( \text{(or)} ) ( mC_p(T_2 - T_1) )</td>
<td>( mC_p \ln \left( \frac{T_2}{T_1} \right) ) (or) ( mC_n \ln \left( \frac{V_2}{V_1} \right) ) (or) ( mC_n \ln \left( \frac{P_1}{P_2} \right) )</td>
<td>( mR \ln \left( \frac{V_2}{V_1} \right) ) (or) ( mR \ln \left( \frac{P_1}{P_2} \right) )</td>
<td>( \gamma - n \frac{\gamma}{\gamma - 1} \times ) Polytropic sp. heat ( C_n )</td>
</tr>
</tbody>
</table>

Basic Concepts and First Law 1.85
Problem 1.23: In a vessel 10 kg of $O_2$ is heated in a reversible non-flow constant volume process, so that the pressure of $O_2$ is increased two times that of initial value. The initial temperature is $20^\circ C$. Calculate the final temperature, change in internal energy, the change in enthalpy and heat transfer. Take $R = 0.259$ kJ/kg K and $C_v = 0.652$ kJ/kg K for oxygen.

\[ \text{(Nov/Dec 2010 - AU)} \]

Solution:

Given data: Constant Volume process,

\[ m = 10 \text{ kg} ; \quad P_2 = 2 P_1; \quad T_1 = 20^\circ C + 273 = 293 \text{ K} \]
\[ R = 0.259 \text{ kJ/kg K} ; \quad C_v = 0.652 \text{ kJ/kg K} \]
\[ R = C_p - C_v \Rightarrow C_p = R + C_v \]
\[ C_p = 0.259 + 0.652 = 0.911 \text{ kJ/kg K} . \]

To find $T_2$

For Constant Volume process, \[ \frac{P_1}{P_2} = \frac{T_1}{T_2} \]

\[ \frac{293}{2} \Rightarrow T_2 = 586 \text{ K} \]

To find Change in internal energy, $\Delta U$:

\[ \Delta U = mC_v (T_2 - T_1) = 10 \times 0.652 \times (586 - 293) = 1910.36 \text{ kJ} \]

$\Delta U = 1910.36$ kJ

To find Change in enthalpy, $\Delta H$

\[ \Delta H = mC_p (T_2 - T_1) = 10 \times 0.911 \times (586 - 293) = 2669.23 \text{ kJ} \]

For constant volume process, $W = \int Pdv = 0$

From first law of thermodynamics,

\[ Q = W + \Delta U = 0 + \Delta U \]

\[ Q = 1910.36 \text{ kJ} \]
Problem 1.24: A certain ideal gas has $R = 290 \text{ J/kg K}$ and $\gamma = 1.35$
(i) Determine the values of $C_p$ and $C_v$. (ii) The mass of the gas filled in a vessel of 0.5 m$^3$ capacity till the pressure inside becomes 4 bar gauge and the temperature is 27°C (iii) if 40 kJ of heat is given to the vessel when the vessel is closed, determine the resulting temperature and pressure. Take the atmospheric pressure $= 100 \text{ kPa}$. (Nov/Dec 2011 - AU)

Solution

For an ideal gas with $R = 290 \text{ J/kg K}$; $\gamma = 1.35$

(i) Values of $C_p$ and $C_v$

$R = C_p - C_v = 290$

$\gamma = \frac{C_p}{C_v} = 1.35 \Rightarrow C_p = 1.35 C_v$

$1.35 C_v - C_v = 290$

$C_v = \frac{290}{0.35} = 828.57 \text{ J/kgK}$

$C_p = 1.35 C_v = 1118.57 \text{ J/kgK}$

(ii) Mass of gas ($m$)

$PV = mRT$

$m = \frac{PV}{RT} = \frac{400 \times 0.5}{0.29 \times 300} = 2.3 \text{ kg}$

(iii) $Q = mC_v (T_2 - T_1)$

$40 = 2.3 \times 0.8286 (T_2 - 300)$

$\therefore$ Resulting temperature $T_2 = 321 \text{ K}$

To find resulting pressure

$\frac{P_1}{T_1} = \frac{P_2}{T_2}$  $[\because V_1 = V_2]$

$4 \frac{P_2}{300} = \frac{P_2}{321} \Rightarrow P_2 = 4.28 \text{ bar}$.
1.44 UNIVERSITY PROBLEMS IN CONSTANT PRESSURE AND CONSTANT VOLUME PROCESS

Problem 1.25: Derive an expression for change in entropy for a constant pressure process. When air temperature changes from 300°C to 100°C, find the change in entropy per kg, if the changes occur by (i) Constant pressure process and (ii) Constant volume process. (Apr/May 2011 - AU)

Solution:

Note:
The derivation for change in entropy for a constant pressure process is already given.

Given data:

\[ T_1 = 300°C = 300 + 273 = 573 \text{ K}; \quad T_2 = 100°C = 100 + 273 = 373 \text{ K} \]

Assume

\[ C_p = 1.005 \text{ kJ/kg K}; \quad C_v = 0.718 \text{ kJ/kg K} \]

To find Change in entropy \( \Delta S \)

(i) \( \Delta S \) For Constant pressure process

\[ \Delta S = mC_p \ln \left( \frac{T_2}{T_1} \right) \]

\[ \Delta S = 1 \times 1.005 \times \ln \left( \frac{373}{573} \right) = -0.4315 \]

\[ \Delta S = -0.4315 \text{ kJ/K} \]

(ii) \( \Delta S \) For Constant volume process

\[ \Delta S = mC_v \ln \left( \frac{T_2}{T_1} \right) \]

\[ \Delta S = 1 \times 0.718 \times \ln \left( \frac{373}{573} \right) = -0.3082 \]

\[ \Delta S = -0.3082 \text{ kJ/K}. \]

Problem 1.26: 1.5 kg of air at 2 bar and 40°C is heated at constant pressure until the volume is doubled. Calculate the work done, the heat transfer and the change in entropy. (Apr/May 2008 - AU)

Solution

Given: \( m = 1.5 \text{ kg}; \quad P_1 = 200 \text{ kPa} \)
\[ T_1 = 40 + 273 = 313 \text{ K}; \text{ constant pressure process.} \]

\[ P_1 = P_2 = 200 \text{ kPa}; \ V_2 = 2V_1. \]

\[ \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \quad \frac{T_2}{T_1} = \frac{V_2}{V_1} \]

\[ \frac{T_2}{T_1} = \frac{2V_1}{V_1} = 2; \]

\[ T_2 = 2T_1 = 2 \times 313 = 626 \text{ K} \]

**To Find** \( W \)

\[ W = mR(T_2 - T_1) = 1.5 \times 0.287(626 - 313) = 134.75 \text{ kJ} \]

[··· \( R = 0.287 \text{ for air} \)]

**To Find** \( Q \)

\[ Q = mC_p(T_2 - T_1) = 1.5 \times 1.005(626 - 313) = 471.85 \text{ kJ} \]

[··· \( C_p = 1.005 \text{ for air} \)]

**To Find** \( \Delta S \)

\[ \Delta S = mC_p \ln \frac{T_2}{T_1} = 1.5 \times 1.005 \ln \left( \frac{626}{313} \right) = 1.045 \text{ kJ/ K} \]

**Problem 1.27:** A stationary mass of a gas is compressed without friction from an initial state of 0.3 m\(^3\) and 0.105 MPa to a final state of 0.15 m\(^3\) and 0.105 MPa, the pressure remaining constant during the process. There is a transfer of 37.6 kJ of heat from the gas during the process. How much does the internal energy of the gas change? (Nov/Dec 2017 - AU)

**Solution**

\[ V_1 = 0.3 \text{ m}^3 \quad V_2 = 0.15 \text{ m}^3 \]

\[ P_1 = 0.105 \text{ MPa} \quad P_2 = 0.105 \text{ MPa} \]

\[ Q = -37.6 \text{ kJ} \]

\[ \Delta U = ? \]
Here, in constant pressure process, 

Work done, \( W = P (V_2 - V_1) \)

\[
= 0.105 \times 10^6 (0.15 - 0.3) = -15750 \text{ J}
\]

\[
= -15.750 \text{ kJ}
\]

(–) sign indicates work is done on the system.

We know that \( Q = \Delta U + W \)

\[
-37.6 = \Delta U - 15.750
\]

\[
\Delta U = -37.6 + 15.750
\]

\[
= -21.85 \text{ kJ}
\]

**Problem 1.28**: 2.5 kg of air is heated at constant pressure of 250 kPa and 100°C until the temperature is 250°C. Find the amount of heat added and change in entropy. \( \text{(Apr/May 2007 - AU)} \)

**Solution**

Given: \( m = 2.5 \text{ kg}; \) constant pressure process

\( P_1 = 250 \text{ kPa}; \) \( T_1 = 100 + 273 = 373 \text{ K}; \)

\( T_2 = 250 + 273 = 523 \text{ K} \)

To Find \( Q \)

\[
Q = mc_p(T_2 - T_1)
\]

\[
= 2.5 \times 1.005(523 - 373)
\]

\[
= 376.875 \text{ kJ}
\]

[\( \text{\`C_p = 1.005 kJ/kg K for air} \)]

To Find \( \Delta S \)

\[
\Delta S = mc_p \ln \frac{T_2}{T_1} = 2.5 \times 1.005 \ln \left( \frac{523}{373} \right)
\]

\[
= 0.8492 \text{ kJ/K}
\]
Problem 1.29: 0.5 kg of air initially at 25°C is heated reversibly at constant pressure until the volume is doubled and is then heated reversibly at constant volume, until the pressure is doubled. For the total path, find the work transfer, the heat transfer and change in entropy. Sketch the processes of air in P-V and T-S diagram.

Solution Given: \( m = 0.5 \text{ kg}; \ T_1 = 25 + 273 = 298 \text{ K} \)

**Process 1-2**

Constant pressure process:

\[ P_1 = P_2 \]

\[ \frac{V_1}{T_1} = \frac{V_2}{T_2} \]

\[ T_2 = 2 \frac{V_2}{V_1} = 2 \quad [\because \ V_2 = 2 \ V_1] \]

\[ T_2 = 2 \ T_1 = 2 \times 298 = 596 \text{ K} \]

To find \( W_{1-2} \)

\[ W_{1-2} = m \ R \ (T_2 - T_1) \]

\[ = 0.5 \times 0.287 \ (596 - 298) \]

\[ = 42.763 \text{ kJ} \]

To find \( Q_{1-2} \)

\[ Q_{1-2} = m \ C_p \ (T_2 - T_1) \]

\[ = 0.5 \times 1.005 \ (596 - 298) \]

\[ = 149.745 \text{ kJ} \]

To find \( (\Delta S)_{1-2} \)

\[ (\Delta S)_{1-2} = m \ C_p \ \ln \left( \frac{T_2}{T_1} \right) \]

\[ = 0.5 \times 1.005 \ln \left( \frac{596}{298} \right) \]

\[ = 0.34831 \text{ kJ/K} \]

**Process 2-3**

Constant Volume Process

\[ V_2 = V_3 \]

\[ P_3 = 2 \ P_2 \]

To find \( W_{2-3} \)

\[ W_{2-3} = 0 \quad \text{for constant volume process.} \]

\[ \frac{T_2}{P_2} = \frac{T_3}{P_3} \]

\[ \frac{T_2}{P_2} = \frac{P_3}{P_2} = 2 \]

\[ T_3 = 2 \ T_2 = 2 \times 596 = 1192 \text{ K} \]

To find \( Q_{2-3} \)

\[ Q = m \ C_v \ (T_3 - T_2) \]

\[ = 0.5 \times 0.718 \ (1192 - 596) \]

\[ = 213.964 \text{ kJ} \]

To find \( (\Delta S)_{2-3} \)

\[ (\Delta S)_{2-3} = m \ C_v \ \ln \left( \frac{T_3}{T_2} \right) \]

\[ = 0.5 \times 0.718 \ln \left( \frac{1192}{596} \right) \]

\[ = 0.24884 \text{ kJ/K} \]

Total Work done \( W_{1-3} = W_{1-2} + W_{2-3} = 42.763 + 0 = 42.763 \text{ kJ} \)
Total heat transfer: \( Q_{2-3} = Q_{1-2} + Q_{2-3} = 149.745 + 213.964 = 363.709 \text{ kJ} \)

Total \((\Delta S)_{1-3} = (\Delta S)_{1-2} + (\Delta S)_{2-3} = 0.34831 + 0.24884 = 0.59715 \text{ kJ/K} \)

1.45 UNIVERSITY PROBLEMS IN ISOTHERMAL PROCESS

**Problem 1.30:** A gas has a specific volume of 0.36 \(m^3/kg\) measured at temperature 15°C and pressure of 1 bar. What would be the density of gas at a pressure of 0.6 bar and same temperature? (Nov/Dec 2010 - AU)

**Solution:** Given: \( v_1 = 0.36 \text{ m}^3/\text{kg}; T_1 = 15 + 273 = 288 \text{ K}; \)

\( P_1 = 1 \text{ bar} = 100 \text{ kPa} \)

\( P_2 = 0.6 \text{ bar} = 60 \text{ kPa}; T_2 = T_1; \) so it is isothermal process.

So, \( P_1 v_1 = P_2 v_2 = \text{constant} \)

\( 100 \times 0.36 = 60 \times v_2 \implies v_2 = 0.6 \text{ m}^3/\text{kg} \)

\[ \rho_2 = \frac{1}{v_2} = \frac{1}{0.6} = 1.6667 \text{ kg/m}^3 \]

Density of a gas at pressure \( \rho_2 = 1.6667 \text{ kg/m}^3 \)
Problem 1.31: A gas occupies 0.3 m$^3$ at 2 bar. It executes a cycle consisting of processes: (1)1-2; constant pressure with work interaction of 15 kJ. (2) 2-3; compression process which follows the law $PV = \text{Constant}$ and $U_3 = U_2$ (3) 3-1; constant volume process, and change in internal energy is 40 kJ. Neglect change in KE and PE. Draw PV diagram for the process and determine network transfer for the cycle. Also show that first law is obeyed by the cycle. (FAQ)

Solution:

Given data: $V_1 = 0.3 \text{ m}^3$

\[
\begin{align*}
P_1 & = 2 \text{ bar} \\
W_{1-2} & = 15 \text{ kJ} \\
U_3 & = U_2 \\
U_1 - U_3 & = 40 \text{ kJ}
\end{align*}
\]

Process 1-2 (constant pressure process) from first law of thermodynamics

\[
Q_{1-2} = (U_2 - U_1) + W_{1-2} \ldots (i)
\]

We know that, $\int dU = 0$

\[
\therefore (U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0
\]

Since $U_3 = U_2$ and $U_1 - U_3 = 40 \text{ kJ}$

\[
U_2 - U_1 + 0 + 40 = 0
\]

\[
\therefore U_2 - U_1 = -40 \text{ kJ}
\]

equation (i) $\Rightarrow Q_{1-2} = -40 + 15 = -25 \text{ kJ}$

Process 1-3 (constant volume process)

For constant volume process, $W_{3-1} = 0$

\[
Q_{3-1} = (U_1 - U_3) - 0
\]

\[
= 40 \text{ kJ}
\]
Process 2-3 (constant temperature process)

\[ Q_{2-3} = (U_3 - U_2) + W_{2-3} \]

\[ Q_{2-3} = 0 + W_{2-3} \]

\[ \therefore Q_{2-3} = W_{2-3} = P_2V_2\ln \left( \frac{V_3}{V_2} \right) \]

\[ [P_2 = P_1 = 200 \text{ kPa}] \]

We know that, \( W_{1-2} = P_1(V_2 - V_1) \)

\[ 15 = 200(V_2 - 0.3) \]

\[ \Rightarrow V_2 = 0.375 \text{ m}^3 \]

\[ W_{2-3} = 200 \times 0.375 \ln \left( \frac{0.3}{0.375} \right) = -16.74 \text{ kJ} \]

\[ \therefore \text{Net work done} = 15 - 16.74 + 0 = -1.74 \text{ kJ} \]

Problem 1.32: A perfect gas for which ratio of specific heat is 1.4 occupies a volume of 0.3 m³ at 100 kPa and 27°C. The gas is compressed to 0.06 m³. Find the heat transfer during compression for the following methods of compression work. (i) PV = Constant (ii) isentropic. Compare the compression work. (Molecular weight of gas is 28) (Nov/Dec 2007 - AU)

Solution: Given \( \gamma = 1.4; \ V_1 = 0.3 \text{ m}^3; \ P_1 = 100 \text{ K Pa}; \)

\[ T_1 = 27 + 273 = 300 \text{ K}; \ V_2 = 0.06 \text{ m}^3 \]

(i) **PV = Constant (Isothermal Process)**

To find \( Q \)

\[ Q = W \] for isothermal process

\[ Q = W = PV\ln \left( \frac{V_2}{V_1} \right) = P_1V_1\ln \left( \frac{V_2}{V_1} \right) \]

\[ = 100 \times 0.3 \ln \left( \frac{0.06}{0.3} \right) = -48.283 \text{ kJ} \]

(ii) **Isentropic process**: \( PV^\gamma = C; \ P_1V_1^\gamma = P_2V_2^\gamma \)

\[ 100 \times 0.3^{1.4} = P_2 \times 0.06^{1.4} \Rightarrow P_2 = 951.83 \text{ kPa} \]

\[ Q = 0 \]

[\( \therefore \text{Isentropic process = Reversible Adiabatic process; } Q = 0 \)]
\[ W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{100 \times 0.3 - 951.83 \times 0.06}{1.4 - 1} = -67.774 \text{kJ} \]

In Isothermal process, minimum work (48.283 \text{kJ}) is required to compress the gas.

\[ \mu = \text{molecular wt} = 28 \]
\[ \bar{R} = \text{Universal gas constant} = 8.3143 \frac{\text{kJ}}{\text{kg mol K}} \]
\[ R = \text{Characteristic gas constant} = \frac{\bar{R}}{\mu} = \frac{8.3143}{28} = 0.29694 \frac{\text{kJ}}{\text{kg K}} \]

For Isothermal Process \( T_1 = T_2 \)

To find \( Q \)
\[ Q = m R T_1 \ln \left( \frac{V_2}{V_1} \right) \]
\[ P_1 V_1 = m R T_1 \]
\[ 100 \times 0.3 = m \times 0.29694 \times 300 \Rightarrow m = 0.33677 \text{ kg} \]
\[ Q = 0.33677 \times 0.29694 \times 300 \ln \left( \frac{0.06}{0.3} \right) = 48.283 \text{ kJ} = W \]

For Isentropic process
\[ Q = 0; \ W = \frac{m R (T_1 - T_2)}{\gamma - 1} \]
\[ \frac{T_1}{T_1} = \left( \frac{V_1}{V_2} \right)^{\gamma - 1} ; \ T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma - 1} \]
\[ T_2 = 300 \left( \frac{0.3}{0.06} \right)^{0.4} = 571.0962 \text{ K} \]
\[ W = \frac{0.33677 \times 0.29694 \times (300 - 571.0962)}{0.4} = -67.774 \text{ kJ} \]

Isothermal process requires very less compression work (48.283 \text{kJ})
Problem 1.33  Three grams of nitrogen gas at 6 atm and 160°C is expanded
adiabatically to double its initial volume, then compressed at constant pressure
to its initial volume and then compressed again at constant volume to its initial
state. Calculate the net work done on the gas. Draw the $p-V$ diagram for the
process. Specific heat ratio of nitrogen is 1.4. (Nov/Dec 2014 - AU)

Solution

\[ m = 0.003 \text{ kg} ; \quad P_1 = 6 \times 1.01325 = 6.0795 \text{ bar} \]

\[ T_1 = 160 + 273 = 433 \text{ K} \]

\[ V_2 = 2V_1 \]

\[ P_2 = P_3 ; \quad V_3 = V_1 \]

The $p-V$ diagram for the process is shown in figure below.

For nitrogen, the gas constant

\[ R = \frac{8314}{28} = 296.93 \text{ J/kgK} \]

we know

\[ P_1V_1 = mRT_1 \]

\[ \Rightarrow V_1 = \frac{mRT_1}{P_1} = \frac{0.003 \times 0.2963 \times 433}{6.0795 \times 10^2} \]

\[ = 6.34 \times 10^{-4} \text{ m}^3 \]

\[ V_2 = 2V_1 = 1.269 \times 10^{-3} \text{ m}^3 \]

For isentropic process

\[ P_1V_1^\gamma = P_2V_2^\gamma \]

\[ \Rightarrow P_2 = P_1 \left[ \frac{V_1}{V_2} \right]^\gamma = 6.0795 \left( \frac{1}{2} \right)^{1.4} = 2.303 \text{ bar} \]

The work done during the process (1 - 2) is given by

\[ W_{1-2} = \frac{P_2V_2 - P_1V_1}{1 - \gamma} = \frac{230.3 \times 1.269 \times 10^{-3} - 607.95 \times 6.344 \times 10^{-4}}{1 - 1.4} \]

\[ W_{1-2} = 0.233 \text{ kJ} \]
The work done during the process 2-3

\[ W_{2-3} = \int_{2}^{3} Pdv = P_2 (V_3 - V_2) = 230.3 \times (0.6344 - 1.269) \times 10^{-3} \]

\[ = -0.14614 \text{ kJ} \]

The work done during the process 3-1

\[ W_{3-1} = 0, \text{ since } dV = 0 \]

Thus the total work done on the gas is

\[ W = W_{1-2} + W_{2-3} + W_{3-1} = 0.233 - 0.14614 + 0 \]

\[ = 0.087 \text{ kJ} \]

**Problem 1.34:** Air expands from 11 bar at 550° C to a pressure of 3 bar adiabatically. Determine temperature at the end of expansion and work done. Find also the change in entropy.

**Solution:**

**Given data:**

\[ P_1 = 11 \text{ bar} = 1100 \text{ kN/m}^2 ; \]
\[ T_1 = 550° \text{ C} + 273 = 823 \text{ K} \]
\[ P_2 = 3 \text{ bar} = 300 \text{ kN/m}^2 ; \text{ adiabatic process (given)} \]

Take Reversible Adiabatic process

i.e., Isentropic process

**To find** \( T_2 \)

The \( P, V \& T \) relations for reversible adiabatic process

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \]

\[ T_2 = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \times T_1 = \left( \frac{300}{1100} \right)^{\frac{1.4 - 1}{1.4}} \times 823 = 567.779 \]

\[ T_2 = 567.78 \text{ K} \]

**To find** \( \Delta S \)

Change in entropy \( \Delta S = 0 \) for reversible adiabatic process. (Isentropic process)
**Problem 1.35:** A system contains 0.2 m$^3$ of a gas at a pressure of 4 bar and 150°C. It is expanded **adiabatically** till the pressure falls to 1 bar. The gas is then heated at a **constant pressure** till its enthalpy increases by 100 kJ. Determine the total work done. Take $C_p = 1$ kJ/kgK and $C_v = 0.714$ kJ/kgK.

**(Nov/Dec 2011 - AU)**

**Solution**

$V_1 = 0.2$ m$^3$ ; $P_1 = 4$ bar ; $T_1 = 150^\circ$C = 423 K ; $P_2 = 1$ bar = $P_3$.

$C_p = 1$ kJ/kgK ; $H_3 - H_2 = 100$ kJ ; $C_v = 0.714$ kJ/kgK

$R = C_p - C_v = 1 - 0.714 = 0.286$ kJ/kgK

\[
\gamma = \frac{C_p}{C_v} = \frac{1}{0.714} = 1.4
\]

Since the process 1-2 is adiabatic,

\[
\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}
\]

\[
T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 423 \left(\frac{1}{4}\right)^{0.4} = 284.66 \text{ K.}
\]

Work done

\[
W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{mR (T_1 - T_2)}{\gamma - 1}
\]
\[ P_1 V_1 = mRT_1 \]

\[ 400 \times 0.2 = m \times 0.286 \times 423 \]

\[ m = 0.66 \text{ kg} \]

\[ \therefore W_{1-2} = \frac{0.66 \times 0.286 \times (423 - 284.66)}{0.4} = 65.41 \text{ kJ} \]

\[ Q_{2-3} = \Delta H = 100 \text{ kJ} = mC_p (T_3 - T_2) \]

\[ T_3 - 284.66 = \frac{100}{0.66 \times 1} \]

\[ T_3 = 436.175 \text{ K} \]

\[ W_{2-3} = mR (T_3 - T_2) \]

\[ = 0.66 \times 0.286 (436.175 - 284.66) = 28.6 \text{ kJ} \]

\[ \therefore \text{ Total work done} = W_{1-2} + W_{2-3} = 65.41 + 28.6 = 94.01 \text{ kJ} \]

**Problem 1.36:** A gas contained in a cylinder is compressed from 1 MPa and 0.05 m³ to 2 MPa. Compression is governed by PV^{1.4} constant. Internal energy of gas is given by \( U = 7.5 \text{ PV} \), where \( P \) is pressure in kPa and \( V \) is volume in m³. Determine heat, work and change in internal energy assuming compression process to be quasistatic. Also find out work interaction, if the 180 kJ of heat is transferred to system between same states. Also explain why it is different from above? *(Apr/May 2011 - AU)*

**Given**

\( P_1 = 1 \text{ MPa}; P_2 = 2 \text{ MPa}; V_1 = 0.05 \text{ m}^3; Q = 180 \text{ kJ}; U = 7.5 \text{ PV} \)

(a) \( U = 7.5 \text{ PV} \)

\( \Delta U = U_2 - U_1 = 7.5 (P_2 V_2 - P_1 V_1) \)

Now \( P_1 V_1^{1.4} = P_2 V_2^{1.4} \)

\[ V_2 = V_1 \left( \frac{P_1}{P_2} \right)^{\frac{1}{1.4}} = 0.05 \left( \frac{1}{2} \right)^{\frac{1}{1.4}} \]

\[ V_2 = 0.03 \text{ m}^3 \]

\[ \therefore \Delta U = 7.5 (2 \times 10^3 \times 0.03 - 1 \times 10^3 \times 0.05) \]

\[ \Delta U = 75 \text{ kJ} \]
for a quasi static process

\[ W = \int PdV = P_1 V_1 - P_2 V_2 \]

\[ \frac{\gamma - 1}{1.4 - 1} \]

\[ W = -25 \text{ kJ} \]

\[ Q = \Delta U + W = 75 - 25 \]

\[ Q = 50 \text{ kJ} \]

(b) Here \( Q = 180 \text{ kJ} \)

Since the end states are the same, \( \Delta U \) would remain the same

\[ W = Q - \Delta U \]

\[ = 180 - 75 \]

\[ W = 105 \text{ kJ} \]

The work is not equal since the process is not quasi-static.

**Problem 1.37:** Evaluate the non-flow work of gas and internal energy while it undergoes a reversible adiabatic process \( PV^{1.4} = C \). The initial pressure and volume are 3 bar and 4 m\(^3\) and final volume is 20 m\(^3\). \( C_p = 1.005 \text{ kJ/kg K} \). (Apr/May 2004 - AU)

**Solution**

**Given:** Reversible adiabatic process

\[ P V^{1.4} = C; \gamma = 1.4; P_1 = 3 \text{ bar} = 300 \text{ kPa}; V_1 = 4 \text{ m}^3; \]

\[ V_2 = 20 \text{ m}^3; C_p = 1.005; \text{ So, } R = 0.287 \text{ kJ/kg K} \]

**To Find \( W \)**

\[ W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \]

\[ P_1 V_1^\prime = P_2 V_2^\prime \]

\[ 300 \times 4^{1.4} = P_2 \times 20^{1.4} \]

\[ P_2 = 31.518 \text{ kPa} \]

So, \( W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{300 \times 4 - 31.518 \times 20}{1.4 - 1} = 1424.083 \text{ kJ} \)
To Find $\Delta U$

$$\Delta U = mC_v(T_2 - T_1)$$

(or) According to first law

$$Q = \Delta U + W$$

$$0 = \Delta U + W$$

[$\because Q = 0$ for reversible adiabatic process]

$$\Delta U = -W$$

So, $\Delta U = -1424.083$ kJ

1.47 UNIVERSITY PROBLEMS IN POLYTROPIC PROCESS

Problem 1.38: 0.5 m$^3$ of a perfect gas is initially at a pressure of 1 bar and at a temperature of 25°C. It is compressed according to the law $PV^{1.3} = C$ until the volume is 0.1 m$^3$. Determine the heat transfer during the process. Take $R = 0.287$ kJ/kg K and $C_p = 1.005$ kJ/kg K

Solution:

Given data:

$PV^{1.3} = C$ ; $n = 1.3$ ; $T_1 = 25°C + 273 = 298 K$

$P_1 = 1$ bar $= 100$ kN/m$^2$ ; $V_1 = 0.5$ m$^3$

$V_2 = 0.1$ m$^3$ ; $C_p = 1.005$ kJ/kg K ;

$R = 0.287$ kJ/kg K

To find $m$

$$P_1V_1 = mRT_1$$

$$m = \frac{P_1V_1}{RT_1} = \frac{100 \times 0.5}{0.287 \times 298} = 0.5846 \text{ kg}$$

For polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} \Rightarrow T_2 = \left(\frac{V_1}{V_2}\right)^{n-1} \times T_1$$

$$T_2 = \left(\frac{0.5}{0.1}\right)^{1.3-1} \times 298 = 482.96 \text{ K}$$

$T_2 = 482.96 \text{ K}.$
To find Heat transfer $Q$

Work done $W$

$$W = \frac{mR (T_2 - T_1)}{n - 1}$$

$$W = \frac{0.5846 \times 0.287(298 - 482.96)}{1.3 - 1}$$

$W = -103.44 \text{ kJ}$

Heat transferred $Q$

$$Q = W \times \left( \frac{\gamma - n}{\gamma - 1} \right) = -103.44 \times \left( \frac{1.4 - 1.3}{1.4 - 1} \right)$$

$Q = -25.86 \text{ kJ}$

To find Heat transfer $Q$

$$C = C \left( \frac{\gamma - n}{1 - n} \right)$$

$$= 0.718 \left( \frac{1.4 - 1.3}{1 - 1.3} \right)$$

$= -0.2393$

$Q = mc (T_2 - T_1)$

$= 0.5846 (-0.2393) (482.96 - 298) = -25.88 \text{ kJ}$

$Q = -25.88 \text{ kJ}$

Problem 1.39: 1 kg of air is compressed according to the law $PV^{1.25} = C$ from 1 bar and $15^\circ \text{C}$ to 17 bar. Calculate the change in entropy. $C_p = 1.005 \text{ kJ/Kg K}$ and $C_v = 0.72 \text{ kJ/kg K}$

Solution:

Given data: $m = 1 \text{ kg}$; $PV^{1.25} = C$; $n = 1.25$

$P_1 = 1 \text{ bar} = 100 \text{ kN/m}^2$; $P_2 = 17 \text{ bar} = 1700 \text{ kN/m}^2$

$T_1 = 15^\circ \text{C} + 273 = 288 \text{ K}; C_p = 1.005 \text{ kJ/kg K}; C_v = 0.72 \text{ kJ/kg K}$

$R = C_p - C_v = 1.005 - 0.72 = 0.285 \text{ kJ/kg K}$

For polytropic process,

$$T_2 = \left( \frac{P_2}{P_1} \right)^{n \frac{n-1}{n}} \times T_1$$

$$T_2 = \left( \frac{1700}{100} \right)^{1.25 - 1} \times 288 = 507.55$$

$T_2 = 507.55 \text{ K}$

To find Change in entropy $\Delta S$

$$\gamma = \frac{C_p}{C_v} = \frac{1.005}{0.72} = 1.396$$
\[ C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) = 0.72 \left( \frac{1.396 - 1.25}{1 - 1.25} \right) = -0.42 \]

\[ \Delta S = m \, C_n \ln \left( \frac{T_2}{T_1} \right) = 1 \times (-0.42) \ln \left( \frac{507.55}{288} \right) = -0.238 \text{ kJ/K} \]

**Problem 1.40:** Calculate the change in internal energy, heat transfer and change in enthalpy for 0.5 kg of air expanding according to law \( PV^{1.2} = C \) from 1 MPa and 300°C to 100 kPa. What will be the work done by air during expansion? (Nov/Dec 2009 - AU)

**Solution**

Given: \( m = 0.5 \text{ kg}; PV^{1.2} = C \) [Polytropic process]; \( P_1 = 1000 \text{ kPa}; \)
\( T_1 = 300 + 273 = 573 \text{ K}; P_2 = 100 \text{ kPa}. \)

To Find \( \Delta U \)

\[ \Delta U = mC_v(T_2 - T_1); \quad [C_v = 0.718 \text{ kJ/kg K}] \text{ for air} \]

\[ T_2 = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}}; \quad T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \]

\[ T_2 = 390.4 \text{ K} \]

\[ \Delta U = mC_v(T_2 - T_1) = 0.5 \times 0.718(390.4 - 573) \]
\[ = -65.5605 \text{ kJ} \quad [\text{i.e. Internal energy decreases}] \]

To Find \( Q \& W \)

\[ W = mR(T_1 - T_2) = \frac{0.5 \times 0.287(573 - 390.4)}{0.2} = 131.0155 \text{ kJ} \]

\[ C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) = 0.718 \times \frac{(1.4 - 1.2)}{(1 - 1.2)} = -0.718 \]

\[ Q = mC_n(T_2 - T_1) = 0.5 \times (-0.718) \times (390.4 - 573) = 65.56 \text{ kJ} \]

To Find \( \Delta H \)

\[ \Delta H = mC_p(T_2 - T_1) = 0.5 \times 1.005(390.4 - 573) = -91.757 \text{ kJ} \quad [\text{i.e. enthalpy decreases}] \]
Problem 1.41: Two kg of air at a pressure of 8 bar occupies a volume of 0.3 m$^3$. If the air expands to a volume of 1.5 m$^3$ according to the law $pV^{1.35} = C$, Calculate the work done and change in entropy during the process.

(Apr/May 2010 - AU)

Solution

Given: $m = 2$ kg; $P_1 = 8$ bar; $V_1 = 0.3$ m$^3$; $V_2 = 1.5$ m$^3$

Polytropic process $pV^{1.35} = C$; i.e $n = 1.35$.

To Find $W$

$$W = \frac{mR(T_1 - T_2)}{n - 1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{n-1} \quad ; \quad T_2 = T_1 \left(\frac{P_2}{P_1}\right)^n$$

$$P_1V_1 = mRT_1$$  

$[ \therefore R = 0.287 \text{ for air }]$

$$8 \times 10^2 \times 0.3 = 2 \times 0.287 \times T_1 \Rightarrow T_1 = 418.12 \text{ K}$$

Note

$$P_1V_1^n = P_2V_2^n$$

$$\left(\frac{V_2}{V_1}\right)^n = \frac{P_1}{P_2} \quad ; \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1} \times \frac{T_1}{T_2}$$

Substitute $\frac{P_1}{P_2}$ from (i) in (ii)

$$\left(\frac{V_2}{V_1}\right)^n = \frac{V_2}{V_1} \times \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1} \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$
\[ T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{n-1} \]

\[ = 418.12 \left( \frac{0.3}{1.5} \right)^{0.35} = 238.05 \text{ K} \]

\[ W = mR \left( \frac{T_1 - T_2}{n-1} \right) = \frac{2 \times 0.287(418.12 - 238.05)}{0.35} = 295.321 \text{ kJ} \]

To Find \( \Delta S \)
\[ \Delta S = mC_n \ln \left( \frac{T_2}{T_1} \right) \]

where \( C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) \)

\[ = 0.718 \left( \frac{1.4 - 1.35}{1 - 1.35} \right) = -0.10257 \]

\[ \Delta S = 2 \times (-0.10257) \ln \left( \frac{238.05}{418.12} \right) = 0.1156 \text{ kJ/K} \]

Extra
To Find \( Q \)
\[ Q = mC_n (T_2 - T_1) = 2 \times (-0.10257) (238.05 - 418.12) = 36.94 \text{ kJ} \]

**Problem 1.42: An ideal gas is compressed with a pressure ratio of 10 and the temperature is increased from 30°C to 240°C. Find (i) Polytropic index of compression. (ii) Work of compression per unit mass. (Apr/May 2010 - AU)**

**Solution:** Given: \( \frac{P_2}{P_1} = 10; T_1 = 30 + 273 = 303 \text{ K} \);

\( T_2 = 240 + 273 = 513 \text{ K}; m = 1 \text{ kg.} \)

Assume ideal gas = air.

To Find \( n \) (polytropic index of compression)
\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \]
513 \div 305 = (10)^{\frac{n-1}{n}} \Rightarrow 1.6931 = (10)^{\frac{n-1}{n}}

Taking \ln on both sides,
\ln 1.6931 = \frac{n-1}{n} \ln 10 \Rightarrow 0.526543 = \frac{n-1}{n} (2.302585)

\frac{n-1}{n} = 0.228675 \Rightarrow n - 1 = 0.228675n

n - 0.228675n = 1 \Rightarrow n (1 - 0.228675) = 1

n = 1.2965

To Find \( W \)

\[ W = \frac{mR(T_1 - T_2)}{n - 1} = \frac{1 \times 0.287(303 - 513)}{0.2965} \]

\[ W = -203.272 \text{ kJ/kg} \]

Problem 1.43: 2 kg of air is expanded from 10 bar and 200°C to 3 bar by the law \( PV^{1.25} = C \). Find the change in entropy and the amount of heat transfer.

(Nov/Dec 2010 - AU)

Solution \( m = 2 \text{ kg}; P_1 = 10 \text{ bar}; T_1 = 200 + 273 = 473 \text{ K}; \)

\( P_2 = 3 \text{ bar}; \ PV^{1.25} = C \). Polytropic process.

To Find \( \Delta S \)

\[ \Delta S = mC_n \ln \frac{T_2}{T_1} \]

where \( C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) (C_n = \text{Polytropic specific heat}) \)

\[ = 0.718 \left( \frac{1.4 - 1.25}{1 - 1.25} \right) \]

\[ = -0.4308 \]

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \text{; } T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \]
\[ T_2 = 473 \left( \frac{3}{10} \right)^{0.25} = 371.78 \text{ K} \]

\[ \Delta S = mC_n \ln \left( \frac{T_2}{T_1} \right) = 2 \times (-0.4308) \ln \left( \frac{371.78}{473} \right) = 0.2075 \text{ kJ/K} \]

To Find \( Q \), \( Q = mC_n (T_2 - T_1) = 2 \times (-0.4308) (371.78 - 473) = 87.2112 \text{ kJ} \)

**Problem 1.44:** Air at 475 kPa and 200°C expands in a non-flow process to 75 kPa as per the law \( PV^{1.2} = C \). Find the work done, heat transfer and change in entropy per kg (Nov/Dec 2009 - AU)

**Solution**

Given: \( P_1 = 475 \text{ kPa}; T_1 = 200 + 273 = 473 \text{ K}; \)

Non-flow process; \( P_2 = 75 \text{ kPa}; PV^{1.2} = C; \)

**Polytropic process:** \( n = 1.2; m = 1 \text{ kg} \)

To Find \( W \)

\[ W = \frac{mR(T_1 - T_2)}{n - 1} \]

\[ T_2 = \left( \frac{P_2}{P_1} \right)^\frac{n-1}{n} ; T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \]

\[ T_2 = 473 \left( \frac{75}{475} \right)^{0.2} = 347.741 \text{ K} \]

\[ W = \frac{1 \times 0.287(473 - 347.741)}{0.2} = 179.747 \text{ kJ/kg} \]

To Find \( Q \)

\[ Q = mC_n(T_2 - T_1) \]

where \( C_n = \text{Polytropic specific heat} = C_v \left( \frac{\gamma - n}{1 - n} \right) \]

\[ = 0.718 \left( \frac{1.4 - 1.2}{1 - 1.2} \right) = -0.718 \]

\[ Q = 1 \times (-0.718) (347.741 - 473) = 89.936 \text{ kJ/kg} \]
To Find $\Delta S$

$$\Delta S = mC_p \ln \frac{T_2}{T_1} = 1 \times (-0.718) \ln \left(\frac{347.741}{473}\right) = 0.2208837 \text{ kJ/kg K}$$

$\Delta S = 220.088 \text{ J/kg K}$

**Problem 1.45:** An ideal gas of molecular weight 30 and specific heat ratio $\gamma = 1.4$ is compressed according to the law $PV^{1.25} = C$ from 1 bar absolute and 27°C to a pressure of 16 bar abs. Calculate the temperature at the end of compression, the heat received or rejected and work done on the gas during the process and change in enthalpy. Assume mass of the gas is 1 kg.

(Apr/May 2005 - AU)

***Solution:***

**Given data:** $\mu = \text{molecular weight} = 30$; $\frac{C_p}{C_v} = \gamma = 1.4$

$PV^{1.25} = C$; So polytropic process

$P_1 = 1 \text{ bar} = 100 \text{ kN/m}^2$; $P_2 = 16 \text{ bar} = 1600 \text{ kN/m}^2$

$T_1 = 27^\circ \text{C} + 273 = 300 \text{ K}$; $m = 1 \text{ kg}$.

**To find $T_2$:**

For polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \times T_1$$

$$T_2 = \left(\frac{1600}{100}\right)^{\frac{1.25-1}{1.25}} \times 300 = 522.33 \text{ K}$$

$T_2 = 522.33 \text{ K}$

**To find $W$:**

Work done for this process $W = \frac{mR(T_1 - T_2)}{n - 1}$

$$\text{Gas constant } R = \frac{\bar{R} \text{ (Universal gas Constant)}}{\mu \text{ (molecular weight)}} = \frac{8.3143}{30} \text{ kJ/kg K} = 0.277 \text{ kJ/kg K}$$
\[ W = \frac{1 \times 0.277 (300 - 522.33)}{1.25 - 1} = -246.34 \]

\[ W = -246.34 \text{ kJ} \]

(\text{\text{-} ve sign indicates that work is done on the system})

\textbf{To find } Q

Heat transferred for this process

\[ Q = \left( \frac{\gamma - n}{\gamma - 1} \right) \times W \]

\[ Q = \left( \frac{1.4 - 1.25}{1.4 - 1} \right) \times (-246.34) = -92.378 \]

\[ Q = -92.378 \text{ kJ} \quad \text{(\text{-} ve sign indicates heat is rejected from the system)} \]

\textbf{To find } \Delta S

Change in entropy (\( \Delta S \))

\[ \Delta S = mC_n \ln \left( \frac{T_2}{T_1} \right) \quad \text{where } C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) \]

\[ C_p = 1.4 \ C_v \]

\[ C_p - C_v = R \Rightarrow 1.4 \ C_v - C_v = 0.277 \]

\[ C_v = 0.6925 \text{ kJ/kg K} \]

\[ C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) = \text{polytropic specific heat} \]

\[ = 0.6925 \left( \frac{1.4 - 1.25}{1 - 1.25} \right) = -0.4155 \text{ kJ/kg K} \]

\[ \Delta S = mC_n \ln \left( \frac{T_2}{T_1} \right) \]

\[ \Delta S = 1 \times (-0.4155) \ln \left( \frac{522.33}{300} \right) = -0.23 \text{ kJ/K} \]

(\text{\text{-} sign indicates that entropy is decreased during this process).}
Problem 1.46: 2 kg of air at 32°C is expanded in a closed system process following $PV^{1.25} = C$ until the pressure is halved. Compute (i) Change in internal energy (ii) Change in entropy (iii) Work done.

Solution:

Given data:

$m = 2 \text{ kg} ; \ T_1 = 32^\circ \text{C} = 32 + 273 = 305 \text{ K}$

$PV^{1.25} = \text{Constant} \Rightarrow n = 1.25 \text{ So Polytropic process.}$

$P_2 = \frac{P_1}{2} ; \ C_n = \text{Polytropic specific heat}$

$C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) = 0.718 \left( \frac{1.4 - 1.25}{1 - 1.25} \right) = -0.4308$

For polytropic process

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{n-1} \Rightarrow T_2 = \left( \frac{P_2}{P_1} \right)^{n-1} \times T_1$$

$$T_2 = \left( \frac{P_1}{P_2} \right)^{1.25} = \left( \frac{2}{1} \right)^{1.25} \times 305 = 265.5 \text{ K}$$

To find Change in internal energy $\Delta U$

$$\Delta U = mC_v (T_2 - T_1) = 2 \times 0.718 (265.5 - 305) = -56.72$$

$\Delta U = -56.72 \text{ kJ} \text{ (ve sign indicates that internal energy of system decreases.)}$

To find Change in entropy: $\Delta S$

$$\Delta S = mC_n \ln \left( \frac{T_2}{T_1} \right)$$

where, $C_n = C_v \frac{\gamma - n}{1 - n} = \text{polytropic specific heat.} = -0.4308$

$$\Delta S = 2 \times [-0.4308 \ln \left( \frac{265.5}{305} \right) = 0.1195 \text{ kJ/K}$$

$\Delta S = 0.1195 \text{ kJ/K}$
To find Work done \( W \)

\[
W = \frac{mR (T_1 - T_2)}{n - 1}
\]

\[
W = \frac{2 \times 0.287 \times (305 - 265.5)}{1.25 - 1} = 90.692
\]

\( W = 90.692 \text{ kJ} \)

**Problem 1.47:** 2 kg of hot gases \( (C_p = 1100 \text{ J/kgK} \) and \( \gamma = 1.33 \) expands in a cylinder from 500 kPa, 600°C to 100 kPa. The polytropic index is 1.25. Find the work done, heat transfer and change in internal energy and entropy.

**(Nov/Dec 2007 - AU)**

**Solution:**

**Given data:**

\[
m = 2 \text{ kg} ; \quad C_p = 1100 \text{ J/kgK} = 1.1 \text{ kJ/kgK} ;
\]

\[
\frac{C_p}{C_v} = \gamma = 1.33 ; \quad P_1 = 500 \text{ kPa} = 500 \text{ kN/m}^2 ;
\]

\[
T_1 = 600^\circ \text{C} = 600 + 273 = 873 \text{ K} ; \quad P_2 = 100 \text{ kPa} = 100 \text{ kN/m}^2 ; \quad n = 1.25
\]

\[
\frac{C_p}{C_v} = 1.33 \Rightarrow C_v = \frac{C_p}{1.33} = \frac{1.1}{1.33} = 0.827 \text{ kJ/kgK} \quad \left[ \therefore P_2 = \frac{P_1}{2} \right]
\]

We know that, \( R = C_p - C_v = 1.1 - 0.827 = 0.273 \text{ kJ/kgK} \)

Polytropic specific heat \( C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) \)

\[
= 0.827 \left( \frac{1.33 - 1.25}{1 - 1.25} \right)
\]

\[
C_n = -0.26464
\]

**To find Work done for polytropic process \( W \)**

**To find \( T_2 \)**

For polytropic process \( \therefore \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \Rightarrow T_2 = \left( \frac{P_2}{P_1} \right)^{\frac{1.25 - 1}{1.25}} \times 873 \)

\[
T_2 = 632.73 \text{ K}
\]
For polytropic process
\[ W = \frac{mR(T_1 - T_2)}{n - 1} \Rightarrow W = \frac{2 \times 0.273 \times (873 - 632.73)}{1.25 - 1} \]
\[ W = 524.75 \text{ kJ} \]

Heat transfer
\[ Q = \left( \frac{\gamma - n}{\gamma - 1} \right) \times W \Rightarrow Q = \left( \frac{1.33 - 1.25}{1.33 - 1} \right) \times 524.75 = 127.21 \text{ kJ} \]
\[ Q = 127.21 \text{ kJ} \]

To find Change in internal energy \((\Delta U)\)
\[ \Delta U = mC_v(T_2 - T_1) = 2 \times 0.827 \times (632.73 - 873) = -397.41 \text{ kJ} \]

\[ \Delta U = -397.41 \text{ kJ} \]

To find Change in entropy \((\Delta S)\)
\[ \Delta S = mC_n \ln \left( \frac{T_2}{T_1} \right) \text{ where} \]
\[ C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) = -0.26464 \]
\[ \Delta S = 2 \times [-0.26464 \ln \left( \frac{632.73}{873} \right)] = 0.1704 \text{ kJ/K} \]

Problem 1.48: 1.6 kg of air is compressed according to the law \(PV^{1.3} = C\) from a pressure of 1.2 bar and temperature of 20°C to a pressure of 17.5 bar. Calculate (a) the final volume and temperature (b) work done (c) heat transferred and (d) change in entropy. (Oct. 2010 - AU)

Solution:
Given data:
\[ m = 1.6 \text{ kg} \quad PV^{1.3} = C \quad n = 1.3 \]
\[ P_1 = 1.2 \text{ bar} = 120 \text{ kN/m}^2 \quad T_1 = 20^\circ \text{C} + 273 = 293 \text{ K} \]
\[ P_2 = 17.5 \text{ bar} = 1750 \text{ kN/m}^2 \]
Polytropic process
\[ C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) = 0.718 \left( \frac{1.4 - 1.3}{1 - 1.3} \right) = -0.2393 \]
To find $V_2$ and $T_2$

By general gas equation

\[ P_1 V_1 = mRT_1 \]

\[
V_1 = \frac{mRT_1}{P_1} = \frac{1.6 \times 0.287 \times 293}{120} = 1.121 \text{ m}^3
\]

\[
P_1 V_1^{1.3} = P_2 V_2^{1.3}
\]

\[
\frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{\frac{1}{1.3}} \Rightarrow V_2 = \left( \frac{120}{1750} \right)^{\frac{1}{1.3}} \times 1.121 = 0.1427
\]

\[
V_2 = 0.1427 \text{ m}^3
\]

\[
P_2 V_2 = mRT_2
\]

\[
T_2 = \frac{P_2 V_2}{mR} = \frac{1750 \times 0.1427}{1.6 \times 0.287} = 543.826
\]

\[
T_2 = 543.826 \text{ K}
\]

To find Work done $W$

\[
W = \frac{P_1 V_1 - P_2 V_2}{n - 1}
\]

\[
W = \frac{120 \times 1.121 - 1750 \times 0.1427}{1.3 - 1} = -384.02
\]

\[
W = -384 \text{ kJ}
\]

To find Heat transfer $Q$

\[
Q = \left( \frac{\gamma - n}{\gamma - 1} \right) \times W
\]

\[
Q = \left( \frac{1.4 - 1.3}{1.4 - 1} \right) \times (-384) = -96
\]

\[
Q = -96 \text{ kJ}
\]

[Also $Q = mC_n (T_2 - T_1)$]

\[
= 1.6 \times (-0.2393) (543.826 - 293)
\]

\[
= -96 \text{ kJ}
\]

[\[. \cdot C_n = -0.2393 \]
To find \( \Delta S \)
\[
\Delta S = mC_n \ln \left( \frac{T_2}{T_1} \right)
\]
\[
\Delta S = 1.6 \times (-0.2393) \ln \left( \frac{543.826}{293} \right) = -0.2368 \text{ kJ/K}
\]

**Problem 1.49** A mass of air is initially at 260°C and 700 kPa and occupies 0.028 m³. The air is expanded at constant pressure to 0.084 m³. A polytropic process with \( n = 1.5 \) is then carried out, followed by a constant temperature process. All the processes are reversible.

(i) Sketch the cycle in the \( P-v \) and \( T.S \) planes
(ii) Find the heat received and heat rejected
(iii) Find the efficiency of the cycle. (Apr/May 2003 - AU)

**Solution:**

**Given data:** \( T_1 = 260 + 273 = 533 \text{ K} \); \( P_1 = 700 \text{ kPa} \); \( V_1 = 0.028 \text{ m}^3 \)

1-2 Constant pressure process \( P_1 = P_2 \) \( V_2 = 0.084 \text{ m}^3 \)

Then, Polytropic Process 2-3 \( P V^{1.5} = C \)

Then Constant temperature process 3-1 \( P V = C \) \( T_3 = T_1 \)

(ii) Process 1-2
\[
Q = mC_p (T_2 - T_1)
\]

To find \( m \)
\[
P_1 V_1 = mRT_1
\]
\[
700 \times 0.028 = m \times 0.287 \times 533
\]
\[
m = \frac{700 \times 0.028}{0.287 \times 533} = 0.128
\]
\[
\Rightarrow m = 0.128 \text{ kg}
\]

To find \( T_2 \)
\[
C_p = 1.005 \text{ for air.}
\]
\[
\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{[} \therefore P_1 = P_2 \text{]}
\]

**Diagram:**
- \( P-v \) plot
- \( T-S \) plot
\[ \frac{T_2}{T_1} = \frac{V_2}{V_1} \Rightarrow T_2 = T_1 \times \frac{V_2}{V_1} = 533 \times \frac{0.084}{0.028} 
T_2 = 1599 \text{ } K \]

To find \( Q \)
\[ Q = mC_p (T_2 - T_1) = 0.128 \times 1.005 (1599 - 533) = 137.13 \text{ kJ} \]
Heat received \( Q_{1-2} = 137.13 \text{ kJ} \) in process 1-2

**Process 2-3**
\( PV^{1.5} = C \) Polytropic process
We know \( T_3 = T_1 = 533K \)
\[ Q_{2-3} = mC_n (T_3 - T_2) \]
where \( C_n = C_v \left( \frac{\gamma - n}{1-n} \right) = 0.718 \left( \frac{1.4 - 1.5}{1 - 1.5} \right) = 0.1436 \]
\[ Q_{2-3} = 0.128 \times 0.1436 \times (533 - 1599) = -19.594 \text{ kJ} \]
Heat rejected during **process 2-3** = −19.594 kJ
(Polytropic process) \( Q_{2-3} \)

**Process 3-1**
Constant Temperature process
\[ Q_{3-1} = PV \ln \left( \frac{P_3}{P_1} \right) = mRT_3 \ln \left( \frac{P_3}{P_1} \right) \]

To find \( P_3 \) in process 2-3
\[ \frac{T_3}{T_2} = \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} \Rightarrow \frac{P_3}{P_2} = \left( \frac{T_3}{T_2} \right)^{\frac{n}{n-1}} \]
\[ P_3 = 700 \times \left( \frac{533}{1599} \right)^{\frac{1.4}{0.4}} = 14.968 \]
\[ Q_{3-1} = mR T_3 \ln \left( \frac{P_3}{P_1} \right) = 0.128 \times 0.287 \times 533 \ln \left( \frac{14.968}{700} \right) \]
\[ = -75.29 \text{ kJ} \]

Heat rejected during constant temperature process \( Q_{3-1} = 75.29 \text{ kJ} \)

Total heat received \( = Q_{1-2} = 137.13 \text{ kJ} \)

Total heat rejected \( = Q_{2-3} + Q_{3-1} \)
\[ = -19.594 - 75.29 = -94.883 \]
\[ Q = -94.883 \text{ kJ} \]

(iii) Efficiency of the cycle:
\[ \eta = \frac{\text{Net work done}}{\text{heat received}} \]

Net work done \( = Q_{\text{received}} - Q_{\text{rejected}} = 137.13 - 94.883 \)
\[ = 42.25 \text{ kJ} \]
\[ \eta_{\text{cycle}} = \frac{42.25}{137.13} = 0.3081 \]
\[ = 30.81\% \]

**Problem 1.50:** A piston-cylinder device contains 0.15 kg of air initially at 2 MPa and 350\(^\circ\)C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the network of the cycle.

**(Nov/Dec 2016 - AU)**

**Solution:** Given data:

\[
m = 0.15 \text{ kg}; \quad P_1 = 2 \text{ MPa} = 2 \times 10^3 \text{ kPa}; \quad P_2 = 500 \text{ kPa} \]

\[
T_1 = T_2 = 350 + 273 = 623 \text{ K}; \quad P_1 V_1 = P_2 V_2; \quad n = 1.2 \]

\[
P_3 = P_1 = 2000 \text{ kPa} \]

\[
P_2 V_2^{1.2} = P_3 V_3^{1.2}; \quad P_3 = P_1 \]
Isothermal process 1-2

\[ W_{1-2} = mRT \ln \left( \frac{P_1}{P_2} \right) \]

\[ = 0.15 \times 0.287 \times 623 \ln \left( \frac{2000}{500} \right) \]

\[ W_{1-2} = 37.18 \text{ kJ} \]

Polytropic process 2-3

\[ W_{2-3} = \frac{mR (T_2 - T_3)}{n-1} \]

\[ T_3 = T_2 \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} \Rightarrow T_3 = T_2 \left( \frac{P_3}{P_2} \right)^{\frac{n-1}{n}} \]

\[ T_3 = 623 \left( \frac{2000}{500} \right)^{0.2} = 784.93 \text{ K} \]

\[ W_{2-3} = \frac{0.15 \times 0.287 \times (623 - 784.93)}{0.2} = -34.86 \text{ kJ} \]

\[ W_{2-3} = -34.86 \text{ kJ} \]

Constant pressure process 3-1

\[ W_{3-1} = mR (T_1 - T_3) = 0.15 \times 0.287 \times (623 - 784.93) \]

\[ = -6.971 \]

\[ W_{3-1} = -6.971 \text{ kJ} \]

Network of the cycle \(= W_{1-2} + W_{2-3} + W_{3-1} \)

\[ = 37.18 - 34.86 - 6.971 \]

\[ \sum W = -4.651 \text{ kJ} \]
Problem 1.51: One kg of ideal gas is heated from 18°C to 93°C assuming 
\[ R = 264 \text{ J/kg K} \], ratio of specific heat 1.18 for the gas and heat transfer of 160 kJ find (i) Sp.heats (ii) change in internal energy (iii) change in enthalpy (iv) the work transfer.

Solution: Given 
\[ m = 1 \text{ kg} ;\ T_1 = 18 + 273 = 291 \text{ K} ;\ T_2 = 93 + 273 = 366 \text{ K} \]
\[ R = 0.264 \text{ kJ/kg K} ;\ \gamma = \frac{C_p}{C_v} = 1.18 ;\ Q = 160 \text{ kJ} \]

To find \( C_p \) and \( C_v \)
\[ \frac{C_p}{C_v} = \gamma = 1.18 \]
\[ C_p - C_v = R = 0.264 \]
\[ 1.18C_v - C_v = 0.264 \]
\[ C_v = 1.4667 \text{ kJ/kg K} \]
\[ C_p = C_v + R = 1.4667 + 0.264 = 1.7307 \text{ kJ/kg K} \]

To find \( \Delta U \)
\[ \Delta U = mC_v(T_2 - T_1) = 1 \times 1.4667 \text{ kJ/kg K} \times (366 - 291) = 110.0025 \text{ kJ} \]

To find \( \Delta H \)
\[ \Delta H = mC_p(T_2 - T_1) = 1 \times 1.7307 \text{ kJ/kg K} \times (366 - 291) = 129.8025 \text{ kJ} \]

To find \( W \): According to first law of thermodynamics 
\[ Q = \Delta U + W \]
\[ 160 = 110.0025 + W \]
\[ W = 49.9975 \text{ kJ} = 50 \text{ kJ} \]

Problem 1.52: A mass of 15 kg of air in a piston cylinder device is heated 
from 25°C to 90°C by passing current through a resistance heater inside the 
cylinder. The pressure inside the cylinder is held constant at 300 kPa during 
the process and a heat loss of 60 kJ occurs. Determine the electrical energy 
 supplied in kW-hr and the change in internal energy.

Solution: Given data:
\[ m = 15 \text{ kg} ;\ T_1 = 25 \text{°C} + 273 = 298 \text{ K} ;\ T_2 = 90 \text{°C} + 273 = 363 \text{ K} \]
\[ P_1 = P_2 = 300 \text{ kPa} = 300 \text{ KN/m}^2; \quad Q = -60 \text{ kJ} \]

**Constant pressure process**

To find Electrical Energy supplied

\[ \text{Work done} = \text{Electrical energy supplied} \]

\[ W = mR (T_2 - T_1) \]

\[ W = 15 \times 0.287 \times (363 - 298) = 279.825 \text{ kJ} \quad \left[ \text{R}_{\text{air}} = 0.287 \text{ kJ/kg K} \right] \]

In kW hr = Workdone \times 3600 = 279.825 \times 3600 = 1007.36 \times 10^3

Electrical energy supplied = \(1007.37 \times 10^3\) kW hr.

**Change in internal energy \( \Delta U \)**

\[ \Delta U = Q - W \text{ from 1 law of thermodynamics} \]

\[ = -60 - 279.825 = -339.825 \]

\[ \Delta U = -339.825 \text{ kJ.} \]

**Problem 1.53:** A work done by substance in a reversible non-flow manner is in accordance with \( V = \left(\frac{15}{P}\right) \text{ m}^3 \), where \( P \) is in bar. Evaluate the work done on or by the system as pressure increases from 10 to 100 bar. Indicate whether it is a compression process or expansion process. If the change in internal energy is 500 kJ, calculate the direction and magnitude of heat transfer.

*(Apr/May 2008 - AU)*

**Solution:**

\[ V = \frac{15}{P} \text{ m}^3; \quad \text{So} \quad P = \frac{15}{V} \]

\[ V_1 = \frac{0.15}{P_1} \text{ and } V_2 = \frac{0.15}{P_2} \quad \text{and} \quad \frac{V}{P} = 0.15 \]

\( P_1 \) & \( P_2 \) are in kPa (since \( P \) in 100 kPa)

Also work done, \( W = \int_{V_1}^{V_2} PdV \)

\[ = \int_{V_1}^{V_2} \frac{0.15}{V} dV = 0.15 \ln \frac{V_2}{V_1} \]
Negative sign indicates that work is done on the system, therefore the process is compression. This is also evident from increase in pressure from 10 bar to 100 bar.

We know first law of thermodynamics.

\[ Q = \Delta U + W \]

\[
Q_{1-2} = (U_2 - U_1) + W_{1-2}
\]

\[
Q_{1-2} = 500 + (-0.34538) = 499.65 \text{ kJ}
\]

Here the positive sign indicates heat is supplied to the system and the value is 499.65 kJ.

**Problem 1.54:** A gas of mass 1.5 kg undergoes a quasi-static expansion which follows a relationship \( P = a + bV \), where \( a \) and \( b \) are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are 0.20 m\(^3\) and 1.2 m\(^3\). The specific internal energy of the gas is given by the relation \( u = 1.5 P v - 85 \text{ kJ/kg} \) where \( P \) is in kPa and \( v \) is in m\(^3\)/kg. Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion. (Nov. 2008, Nov/Dec 2012 - AU)

**Solution:**

Quasi-Static expansion

\[
P_1 = 1000 \text{ kPa} ; \ P_2 = 200 \text{ kPa} ; \ V_1 = 0.2 \text{ m}^3 ; \ V_2 = 1.2 \text{ m}^3
\]

Mass = 1.5 kg; \( P = a + bV \); \( P_1 = a + bV_1 \); \( P_2 = a + bV_2 \)

\[
1000 = a + b \ (0.2)
\]

\[
200 = a + b \ (1.2)
\]

\[
800 = -1b \Rightarrow b = -800
\]

Substitute \( b = -800 \) in eqn 1000 = \( a + b \) (0.2)

\[
1000 = a + (-800 \times 0.2) \Rightarrow a = 1160
\]

Change in internal energy \( \Delta U \) during the process
\[ \Delta u = u_2 - u_1 \\
= (1.5 P_2 v_2 - 85) - (1.5 P_1 v_1 - 85) \\
= 1.5 (P_2 v_2 - P_1 v_1) \]

\[ v_2 = \frac{V_2}{1.5} = \frac{1.2}{1.5} = 0.8 \text{ m}^3/\text{kg} \]

\[ v_1 = \frac{V_1}{1.5} = \frac{0.2}{1.5} = 0.1333 \text{ m}^3/\text{kg} \]

\[ \Delta u = 1.5 (200 \times 0.8 - 1000 \times 0.1333) = 1.5 (160 - 133.33) \]

\[ \Delta u = 40.005 \text{ kJ/kg} \]

\[ \Delta U = m \Delta u = 1.5 \times 40 = 60 \text{ kJ} \]

Work transfer during the process

\[ W_{1 \rightarrow 2} = \int_{V_1}^{V_2} P \, dV \\
= \int_{V_1}^{V_2} (a + bV) \, dV = \left[ aV + \frac{bV^2}{2} \right]_{V_1}^{V_2} \\
= a (V_2 - V_1) + b \left( \frac{V_2^2 - V_1^2}{2} \right) \]

\[ W_{1 \rightarrow 2} = 1160 \times (1.2 - 0.2) - 800 \times \left( \frac{1.2^2 - 0.2^2}{2} \right) \]

\[ = 1160 - 560 = 600 \text{ kJ} \]

\[ Q_{1 \rightarrow 2} = (U_2 - U_1) + W_{1 \rightarrow 2} \\
= 60 + 600 \]

\[ Q_{1 \rightarrow 2} = 660 \text{ kJ} \]

Maximum Internal Energy of the gas \( = U_2 \)

\[ U_2 = m (1.5 P_2 v_2 - 85) = 1.5 (1.5 \times 200 \times 0.8 - 85) = 232.5 \text{ kJ} \]
Problem 1.55: A fluid is confined in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is a linear function of the volume \( P = a + bV \). The internal energy of the fluid is given by \( U = (34 + 3.15 PV) \) where \( U \) is in kJ, \( P \) in kPa and \( V \) in cubic meter. If the fluid changes from an initial state of 170 kPa, 0.03 m\(^3\) to a final state of 400 kPa, 0.06 m\(^3\), with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer. (Nov/Dec 2012 - AU)

Solution

The change in the internal energy of the fluid during the process.
\[
U_2 - U_1 = 3.15 (P_2 V_2 - P_1 V_1) = 315 (4 \times 0.06) - 1.7 \times 0.03
\]
\[
= 315 \times 0.189 = 59.5 \text{ kJ}
\]

Now \( P = a + bV \)
\[
170 = a + b \times 0.03
\]
\[
400 = a + b \times 0.06
\]
From these two equations \( a = -60 \)
\[
b = 7667
\]

Work transfer involved during the process
\[
W_{1-2} = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} (a + bV) dV = a (V_2 - V_1) + b \frac{V_2^2 - V_1^2}{2}
\]
\[
= (V_2 - V_1) \left[ a + \frac{b}{2} (V_1 + V_2) \right] = 0.03 \left[ -60 + \frac{7667}{2} \times 0.09 \right]
\]
\[
= 8.55 \text{ kJ}
\]

Work of 8.55 kJ is done by the system
\[
\therefore \text{ Heat transfer involved is given by}
\]
\[
Q_{1-2} = (U_2 - U_1) + W_{1-2} = 59.5 + 8.55 = 68.05 \text{ kJ}
\]

Problem 1.56: A reciprocating air compressor takes in 2 m\(^3\)/min air at 0.11 MPa, 293 K which it delivers at 1.5 MPa, 384 K to an after cooler where the air is cooled at constant pressure to 298 K. The power absorbed by the compressor is 4.15 kW. Determine the heat transfer in (i) the compressor (ii) the cooler. (Nov/Dec 2009 - AU)

Solution: Given:

Initial volume, \( V_1 = 2 \text{ m}^3/\text{min} \); Initial pressure \( P_1 = 0.11 \text{ MPa} \); \( T_1 = 293 \text{ K} \)
Final pressure \( P_2 = 1.5 \text{ MPa} \); \( T_2 = 384 \text{ K} \); \( T_3 = 298 \text{ K} \); \( W_{1.2} = 4.15 \text{ kW} \)
To find mass flow rate,
\[ P_1 V_1 = \dot{m} R T_1 \]
\[ \therefore \dot{m} = \frac{0.11 \times 10^6 \times \frac{2}{60}}{287 \times 293} = 0.044 \text{ kg/s} \]

As per I Law of thermodynamics,
(i) Heat rejected in compressor
\[ Q_{1-2} = \Delta u + W_{1-2} = \dot{m} C_v (T_2 - T_1) + W_{1-2} \]
\[ = 0.044 \times 0.718 (384 - 293) - 4.15 \]
\[ = - 1.275 \text{ kJ/s (Heat is rejected)} \]

Heat transfer in cooler
\[ Q = \dot{m} C_p (T_3 - T_2) \]
\[ = 0.044 \times 1.005 (298 - 384) \]
\[ = - 3.8 \text{ kJ/s (Heat is rejected)} \]

**Problem 1.57:** One kg of ideal gas is heated from 50°C to 150°C. If \( R = 280 \text{ kJ/kgK} \) and \( \gamma = 1.32 \) for the gas, determine 1. \( C_p \) and \( C_v \), 2. Change in internal energy, 3. Change in enthalpy, 4. Change in flow energy

_May/June 2012 - AU_

**Given data:** \( m = 1; \quad T_1 = 50^\circ \text{C} + 273 = 323 \text{ K}; \quad T_2 = 150^\circ \text{C} + 273 = 423 \text{ K}; \quad R = 0.280 \text{ kJ/kgK} = 280 \text{ J/kgK}; \quad \gamma = 1.32 \)

**Solution**
\[
\begin{align*}
C_p - C_v &= R, \quad \frac{C_p}{C_v} = \gamma \\
C_p &= C_v \gamma \\
C_p &= C_v (1.32) \\
&(1.32) C_v - C_v = 280
\end{align*}
\]

From (1) \( C_p = 875 (1.32) = 1155 \text{ J/kgK} \)

(ii) Change in internal energy
\[ \Delta U = U_2 - U_1 = m C_v (T_2 - T_1) = 1 \times 875 (423 - 323) = 87500 \text{ J} = 87.5 \text{ kJ} \]

(iii) Change in enthalpy
\[ \Delta H = H_2 - H_1 = m C_p (T_2 - T_1) = 1 \times 1155 (423 - 323) = 115500 \text{ J} = 115.5 \text{ kJ} \]
1.49 APPLICATION OF FIRST LAW TO OPEN SYSTEM
(STEADY FLOW PROCESS)

Derive steady flow energy equation. (Nov/Dec 2016, Nov/Dec 2013 - AU)

When there is a mass transfer across the boundary, the system is called as open system. Most of the engineering devices are open system, involving the flow of mass through them.

We can mark an imaginary box (broken line around the open system) and this is called control surface. The volume inside the control surface is known as control volume. The mass of fluid is flowing through this control volume.

Steady flow means that the rate of flow of mass and energy are constant. The properties will have fixed value at particular location and will not alter with time.

\[ \dot{m}_1 = \dot{m}_2 = \dot{m} = \text{mass rate of flow in kg/sec} \]

\[ \dot{m} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2 \]

\[ \dot{m} = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2} \]

where \( \rho = \text{density of fluid} \), \( C = \text{Velocity of fluid} \)

\( v = \text{specific volume} \)

\( A = \text{Area of cross section through which mass flows.} \)

According to I Law, \( Q = \Delta E + W \Rightarrow Q = (E_2 - E_1) + W \)
\[ E_1 + Q = E_2 + W \]

\[ E_1 = \text{energy at position 1 for one Kg of mass} \]

\[ E_1 = P E_1 + K E_1 + \text{Flow work} + u_1 \]

P.E. = Potential energy

K.E. = Kientic energy; Flow work = \( P_1 v_1 \)

\[ u = \text{Internal energy} \]

\[ E_1 = Z_1 g + \frac{C_1^2}{2} + P_1 v_1 + u_1; E_2 = Z_2 g + \frac{C_2^2}{2} + P_2 v_2 + u_2 \]

The steady flow energy equation on mass basis is given below.

\[ Z_1 g + \frac{C_1^2}{2} + (P_1 v_1 + u_1) + Q = Z_2 g + \frac{C_2^2}{2} + (P_2 v_2 + u_2) + W \]

Here all energy values are in J/kg.

The steady flow energy equation for \( m \) (kg/sec) of fluid is given as follows. (Time basis)

\[ m \left[ Z_1 g + \frac{C_1^2}{2} + h_1 \right] + \dot{Q} = m \left[ Z_2 g + \frac{C_2^2}{2} + h_2 \right] + \dot{W} \]

where \( \dot{Q} \) in J/s i.e., Watts

\( \dot{W} \) in J/s i.e., Watts

\[ Z_1 = \text{ht above datum at 1 in m} \]

\[ C_1 = \text{Velocity of fluid at 1 in m/sec} \]

\[ h_1 = \text{enthalpy of fluid at 1 in J/kg} \]

\[ A_1 = \text{Area at inlet in m}^2 \]

Similarly \( Z_2, C_2, h_2 \) and \( A_2 \)

The steady flow energy equation

\[ h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \text{ in J/kg (or)} \]

\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W} \text{ in Watts can be applied for} \]

1. Nozzles and Diffuser
2. Throttle Device
3. Turbine and Compressor
4. Heat exchangers (Boiler, Condenser and evaporator)
Formulae to Remember:

Derive the steady flow energy equation and reduce it for a turbine, pump nozzle and a heat exchanger:

1. The Steady Flow Energy Equation (SFEE) is given by per unit mass

\[ h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \text{ in J/kg (or)} \]

\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W} \text{ in Watts} \]

2. For boiler SFEE is \( Q = h_2 - h_1 \)

3. For condenser \( Q = h_2 - h_1 \)

4. For nozzle SFEE is \( \frac{C_2^2 - C_1^2}{2} = (h_1 - h_2) \) \[ \text{[where } h_1, h_2 \text{ are in J/kg]} \]

   Final velocity \( C_2 = \sqrt{2 (h_1 - h_2)} + C_1^2 \text{ m/s} \)

   If initial velocity is neglected

   \[ C_2 = \sqrt{2 (h_1 - h_2)} \]

   \[ \text{Note: Derivations are given in the respective example problems.} \]

5. For turbine SFEE is \( W = h_1 - h_2 \text{ J/kg} \)

6. For rotary Compressor \( W = h_2 - h_1 \text{ J/kg} \)

7. For Reciprocating Compressor \( W = Q + (h_2 - h_1) \text{ J/kg} \)

**Problem 1.58:** A mass of 0.8333 kg/sec enters the control volume of a steady flow system at 2 bar and 100°C and an elevation of 100 m above the datum. The same mass leaves the control volume at 150 m elevation with a pressure of 10 bar and temperature of 300°C. The entrance velocity is 40 m/sec and the exit velocity is 20 m/sec. During the process 13.8889 kW of heat are transferred to the control volume and the rise in enthalpy is 8 kJ/kg. Calculate the power developed.

**Solution**

**Given:** \( \dot{m} = 0.8333 \text{ kg/sec}; \ P_1 = 2 \text{ Bar}; \ T_1 = 100 + 273 = 372 \text{ K Steady flow;} \)

\( Z_1 = 100 \text{ m; } Z_2 = 150 \text{ m; } C_1 = 40 \text{ m/sec; } C_2 = 20 \text{ m/sec; } P_2 = 10 \text{ bar;} \)

\( T_1 = 300 + 273 = 573 \text{ K} \)

\[ \dot{Q} = 13.8889 \text{ kW} = 13.8889 \times 10^3 \text{ W}; \]
Given: \( m = 10 \text{ kg/min} = 0.167 \text{ kg/s} \); \( P_1 = 1.5 \text{ bar} \)

\[ \rho_1 = 26 \text{ kg/m}^3 \Rightarrow \nu_1 = \frac{1}{26} = 0.038 \text{ m}^3/\text{kg} \] ; \( C_1 = 110 \text{ m/s} \) : \( C_1 = 0.11 \text{ km/s} \)
\[ u_1 = 910 \text{ kJ/kg}; \quad P_2 = 5.5 \text{ bar} \]
\[ \rho_2 = 5.5 \text{ kg/m}^3 \Rightarrow v_2 = \frac{1}{5.5} = 0.1818 \text{ kg/m}^3; \quad C_2 = 190 \text{ m/s} = 0.19 \text{ km/s} \]
\[ u_2 = 710 \text{ kJ/kg}; \quad Q = -55 \text{ kJ/s (heat rejection)}; \quad z_2 - z_1 = 55 \text{ m} \]

(i) Change in enthalpy
\[ \Delta h = h_2 - h_1 = (u_2 - u_1) + (P_2 v_2 - P_1 v_1) \]
\[ \Delta h = (710 - 910) + (550 \times 0.1818 - 150 \times 0.038) \]
\[ h_2 - h_1 = -105.71 \text{ kJ/kg} \]

(ii) Workdone during the process (W)
Writing S.F.E.E.
\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + Q_{1-2} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + W_{1-2} \]
\[ \therefore W_{1-2} = \dot{m} \left[ (h_1 - h_2) + \frac{C_1^2 - C_2^2}{2} + (Z_1 - Z_2) g \right] + Q_{1-2} \]
\[ = 0.167 \left[ 105.785 + \frac{(0.11)^2 - (0.19)^2}{2000} - \frac{55 \times 9.81}{1000} \right] - 55 \]
\[ = -37.42 \text{ kW} \]

**Problem 1.60** A room for four persons has two fans, each fan consuming 0.18 kW power, and three 100 W lamps. Ventilation air at the rate of 80 kg/hr enters with an enthalpy of 84 kJ/kg and leaves with an enthalpy of 59 kJ/kg. If each person puts out heat at the rate of 630 kJ/hr, determine the rate at which heat is to be removed by a room cooler, so that a steady state is maintained in the room.  
(Nov/Dec 2010 - AU)

**Solution:**

**Given:** 4 persons ; 2 fans.

Power consumed by 2 fans = 0.18 kW $\times$ 2 = 0.36 kW

Power consumed by 3 lamps = $3 \times 100 = 300$ watts = 0.3 kW

mass of air = 80 kg / hr $\div \frac{80}{3600} = 0.222 \text{ kg/sec}$
Heat put out by 4 persons
$= 630 \times 4 \frac{\text{kJ}}{3600} \text{ sec} = 0.7 \text{ kJ/ sec} = 0.7 \text{ kW}$

Steady Flow Energy Equation gives

$$\dot{m}(h_1 + \frac{C_1^2}{2} + Z_1g) + \dot{Q} = \dot{m}(h_2 + \frac{C_2^2}{2} + Z_2g) + \dot{W}$$

$Z_1 = Z_2 ; \ C_1 = C_2; \ \dot{W} = 0.36 - 0.3 = -0.66 \text{ kW}$

$$\dot{m}h_1 + \dot{Q} = \dot{m}h_2 + \dot{W}$$

$$\dot{Q} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$$

$$\dot{m}h_1 + \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \dot{m}h_2 + \dot{W}$$

$$0.0222 \times 84 + 0.7 - \dot{Q}_{\text{out}} = 0.0222 \times 59 - 0.66$$

$$\dot{Q}_{\text{out}} = 1.915 \text{ kW}$$

The rate at which the heat is to be removed by a room cooler is 1.92 kW.

1.50 NOZZLE

Nozzle is a device which is used to increase the velocity of fluid at the expense of its pressure drop.

1.50.1 Diffusor:

Diffusor is a device which is used to increase the pressure of fluid at the expense of its kinetic energy (by velocity drop)

Problem 1.61: The velocity and enthalpy of fluid at the inlet of a certain nozzle are 50 m/sec and 2800 kJ/kg respectively. The enthalpy at the exit of nozzle is 2600 kJ/kg. The nozzle is horizontal and insulated so that no heat transfer takes place from it. Find 1. Velocity of fluid at exit of the nozzle,
2. Mass flow rate, if the area at inlet of nozzle is 0.09 m\(^2\) and specific volume is 0.185 m\(^3\)/kg.

3. Exit area of the nozzle, if the specific volume at the exit of the nozzle is 0.495 m\(^3\)/kg.

**Solution**

Applying steady flow energy equation for the nozzle

\[
h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W
\]

[mass basis]

Here \(Z_1 g = Z_2 g\), \(Q = 0\) (Insulated; so no heat transfer)

\(W = 0\) (no work transfer during flow through nozzle)

So the equation becomes

\[
h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \Rightarrow C_2^2 = 2 (h_1 - h_2) + C_1^2
\]

\[
C_2 = \sqrt{2 (h_1 - h_2) + C_1^2}
\]

\[
= \sqrt{2 (2800 \times 10^3 - 2600 \times 10^3) + 50^2} = 634.43 \text{ m/sec}
\]

To find Exit Area \(A_2\) and \(\dot{m}\)

\[
\dot{m} = \text{mass flow rate} = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}
\]

\[
\frac{0.09 \times 50}{0.185} = \frac{A_2 \times 634.43}{0.495} \Rightarrow A_2 = 0.018979 \text{ m}^2
\]

\[
\dot{m} = \frac{0.09 \times 50}{0.185} = 24.324 \text{ kg/sec}
\]

**Problem 1.62:** Air expands by isentropic process through a nozzle from 784 kPa and 220°C to an exit pressure of 98 kPa. Determine the exit velocity and the mass flow rate, if the exit area is 0.0006 m\(^2\).

**Solution**

Given: \(P_1 = 784\) kPa = 7.84 bar

\(T_1 = 220\degree C + 273 = 493\) K; \(P_2 = 98\) kPa = 0.98 bar

\(A_2 = 0.0006\) m\(^2\)

\[
\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \Rightarrow T_2 = 493 \left( \frac{0.98}{7.84} \right)^{0.4} = 272.158 \text{ K}
\]
\[ h_1 = C_p T_1 = 1.005 \times 493 = 495.465 \text{kJ/kg} \]
\[ h_2 = C_p T_2 = 1.005 \times 272.158 = 273.52 \text{kJ/kg} \]

\[ P_2 \nu_2 = RT_2 \; \nu_2 = \frac{0.287 \times 272.158}{98} = 0.797 \text{m}^3/\text{kg} \]

for a Nozzle, the steady flow energy equation.

\[ h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \]

Since the initial velocity is negligible,

\[ h_1 = h_2 + \frac{C_2^2}{2} \]

\[ \Rightarrow C_2 = \sqrt{2000 (h_1 - h_2)} = \sqrt{2000 \times (495.465 - 273.52)} = 666.25 \text{ m/sec} \]

we know

\[ \dot{m} = \rho_2 A_2 C_2 = \frac{1}{v_2} A_2 C_2 \]

\[ \dot{m} = \frac{1}{0.797} \times 0.0006 \times 666.25 = 0.502 \text{ kg/sec} \]

**Problem 1.63:** At the inlet of the nozzle, the enthalpy and velocity of the fluid are 3000 kJ/kg and 50 m/s respectively. There is negligible heat loss from the nozzle. At the outlet of the nozzle enthalpy is 2450 kJ/kg. If the nozzle is horizontal, find the velocity of the fluid at exit. (Nov/Dec 2008 - AU)

**Solution:** Given data: \( h_1 = 3000 \text{ kJ/kg} \); \( C_1 = 50 \text{ m/s} \); \( Q = 0 \); \( Z_1 = Z_2 \); \( h_2 = 2450 \text{ kJ/kg} \); \( W = 0 \)

To find Velocity of the fluid at exit \( (C_2) \)

SFEE \( h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \)

The SFEE for nozzle \( \frac{C_2^2}{2} + h_1 = \frac{C_2^2}{2} + h_2 \) \[ \Rightarrow \cdot \cdot Z_1 = Z_2 \]

\[ C_2 = \sqrt{2(h_1 - h_2) + C_1^2} \]
\[ = \sqrt{2 \times (3000 - 2450) \times 10^3 + 50^2} \]

\[ C_2 = 1050 \text{ m/s} \] \[ \Rightarrow \cdot \times 10^3 \text{ for making } h \text{ in J/kg} \]
1.51 THROTTLING DEVICE

It is the device used to reduce the pressure when its enthalpy remains constant.

When a fluid is passing through restricted passage as shown in fig. then it is throttled.

S.F.E.E (Steady Flow Energy Equation)

\[
h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W
\]

During flow through throttling device,

\[Q = 0; W = 0;\] No change in velocity (\(\therefore C_1 = C_2\))

No change in height (So \(Z_1 = Z_2\))

So the equation becomes \(h_1 = h_2\)

So the enthalpy during throttling process remains constant.
1.52 TURBINE

The turbine is used to produce power. Expansion of working fluid takes place in the turbine and the work is done.

Problem 1.64: A turbine operates under steady flow condition receiving steam with the following data. Pressure $P_1 = 1.2 \text{ MPa}$; Temperature $t_1 = 188 \degree \text{C}$; enthalpy $h_1 = 2785 \text{ kJ/kg}$; Velocity $C_1 = 33.3 \text{ m/sec}$ and elevation $Z_1 = 3 \text{ m}$. The steam leaves the turbine at 100 m/sec velocity, zero elevation, pressure $= 20 \text{ kPa}$, $2512 \text{ kJ/kg}$ of enthalpy. Heat is lost to the surrounding at the rate of $0.27 \text{ kJ/sec}$. Determine the power output of the turbine, if the mass flow rate of steam is $0.42 \text{ kg/sec}$.

Solution

\[ P_1 = 1.2 \text{ MPa} = 1.2 \times 10^6 \text{ Pa} \]
\[ T_1 = 188 + 273 = 461 \text{ K} \]
\[ h_1 = 2785 \text{ kJ/kg} \]
\[ = 2785 \times 10^3 \text{ J/kg} \]
\[ C_1 = 33.3 \text{ m/sec} \]
\[ Z_1 = 3 \text{ m} \]
\[ Q = 0.27 \times 10^3 \text{ J/sec} = 270 \text{ J/sec} \]
\[ \dot{m} = 0.42 \text{ kg/sec} \]

\[ P_2 = 20 \text{ kPa} = 20 \times 10^3 \text{ Pa} \]
\[ T_2 = ? \]
\[ h_2 = 2512 \times 10^3 \text{ J/kg} \]
\[ C_2 = 100 \text{ m/sec} \]
\[ Z_2 = 0 \]
Applying S.F.E.E
\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W} \]
\[ \dot{W} = \dot{m} \left[ (h_1 - h_2) + \frac{(C_1^2 - C_2^2)}{2} + (Z_1 g - Z_2 g) \right] + \dot{Q} \]
\[ = 0.42 \left[ (2785 \times 10^3 - 2512 \times 10^3) + \frac{(33.3^2 - 100^2)}{2} + (3 \times 9.81 - 0) \right] - 270 \]
\[ = 112535 \text{ J/sec} = 112.535 \text{ kW} \]

**Problem 1.65:** In a Steam power station, steam flows steadily through a 0.2m diameter pipeline from the boiler to the turbine. At the boiler end, the steam conditions are found to be \( P = 4 \text{ MPa}; \ t = 400^\circ \text{C}; \ h = 3213.6 \text{ kJ/kg} \) and \( v = 0.073 \text{ m}^3/\text{kg} \). At the turbine end, the conditions are found to be \( P = 35 \text{ MPa}; \ t = 392^\circ \text{C}, \ h = 3202.6 \text{ kJ/kg}, \) \( v = 0.084 \text{ m}^3/\text{kg} \). There is a heat loss of 8.5 kJ/kg from the pipeline. Calculate steam flow state.

**Given:**
\( d_1 = 0.2 \text{ m}, \ P_1 = 4 \text{ MPa}, \ t_1 = 400^\circ \text{C}, \ h_1 = 3213.6 \text{ kJ/kg} \) \( v_1 = 0.073 \text{ m}^3/\text{kg} \)
\( P_2 = 3.5 \text{ MPa}, \ t_2 = 392^\circ \text{C}, \ h_2 = 3202.6 \text{ kJ/kg}, \ v_2 = 0.084 \text{ m}^3/\text{kg} \)
\( Q = -8.5 \text{ kJ/kg} \) (Heat loss)

by S.F.E.E.
\[ h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \]

Applying Continuity equation
\[ \frac{A_1 C_1}{V_1} = \frac{A_2 C_2}{V_2} \]
\[ C_2 = \frac{A_1}{A_2} \times \frac{v_2}{v_1} \times C_1 = \frac{0.084}{0.073} = 1.15C_1 \]

Here \( A_1 = A_2 \) since cross-section of pipeline is same from boiler to turbine

And \( W = 0, \ h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W \)
\[ \frac{C_2^2 - C_1^2}{2} \times 10^{-3} = (h_1 - h_2) + Q \]
\[ = (3213.6 - 3202.6) - 8.5 = 2.5 \text{ kJ/kg} \]
\[ (1.15C_1)^2 - C_1^2 = 5 \times 10^3 \Rightarrow C_1^2 = 15504 \, \text{m}^2/\text{s}^2 \]

\[ C_1 = 124.51 \, \text{m/s} \]

Mass flow rate

\[ \dot{m} = A_1 C_1 \frac{\pi \times 0.22^2}{2} \times 124.51 \]

\[ \dot{m} = 53.6 \, \text{kg/s} \]

**Problem 1.66:** A steam turbine operates under steady flow conditions. It receives steam 7200 kg/hr from the boiler. The steam enters the turbine at enthalpy of 2800 kJ/kg, a velocity of 4000 m/min and an elevation of 4 m. The steam leaves the turbine at an enthalpy of 2000 kJ/kg, a velocity of 800 m/min and an elevation of 1 m. Due to radiation, heat losses from the turbine to the surroundings is 1580 kJ/hr. Calculate the power output of the turbine.

**Solution:** Given data:

\[ \dot{m} = 7200 \, \text{kg/hr} = \frac{7200}{3600} = 2 \, \text{kg/sec} ; \quad h_1 = 2800 \, \text{kJ/kg} ; \]

\[ C_1 = 4000 \, \text{m/min} = 66.67 \, \text{m/s} ; \quad Z_1 = 4 \, \text{m} \]

\[ Z_2 = 1 \, \text{m} ; \quad h_2 = 2000 \, \text{kJ/kg} ; \quad C_2 = 8000 \, \text{m/min} = 133.3 \, \text{m/s} \]

\[ Q = 1580 \, \text{kJ/hr} = 0.438 \, \text{kJ/sec} = 0.438 \, \text{kW} \]

To find Power output of the turbine \( (P) \)

The SFEE for the turbine system is

\[ \dot{m} \left[ gZ_1 + \frac{C_1^2}{2} + h_1 \right] + \dot{Q} = \dot{m} \left[ gZ_2 + \frac{C_2^2}{2} + h_2 \right] + \dot{W} \]

Power \( \dot{W} = \dot{m} \left[ g (Z_1 - Z_2) + \frac{C_1^2 - C_2^2}{2} + (h_1 - h_2) \right] + \dot{Q} \) [Here \( h \) is in J/kg]

\[ \dot{W} = 2 \left[ 9.81 \, (4 - 1) + \frac{(66.67)^2}{2} - \frac{(133.3)^2}{2} + (2800 - 2000) \times 10^3 \right] + 0.438 \times 10^3 \]

\[ \dot{W} = 2 \left[ 29.43 - 6662 + 800 \times 10^3 \right] + 0.438 \times 10^3 \]

\[ = 1587.165 \times 10^3 \, \text{J/s} \]

\[ \dot{W} = 1587.165 \, \text{kW} \]
Problem 1.67: In a turbo machine handling an incompressible fluid with a density of 1000 kg/m³ the conditions of the fluid at the rotor entry and exit are as given below:

<table>
<thead>
<tr>
<th></th>
<th>Inlet</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>1.15 MPa</td>
<td>0.05 MPa</td>
</tr>
<tr>
<td>Velocity</td>
<td>30 m/sec</td>
<td>15.5 m/sec</td>
</tr>
<tr>
<td>Height above datum</td>
<td>10 m</td>
<td>2 m</td>
</tr>
</tbody>
</table>

If the volume flow rate of the fluid is 40 m³/s, estimate the net energy transfer from the fluid as work. (Nov/Dec 2009 - AU)

Solution:

In compressible flow, \( \rho_1 = \rho_2 = 1000 \text{ kg/m}^3 \)

\( \dot{v}_1 = \dot{v}_2 = 40 \text{ m}^3/\text{s} \)

To find mass flow rate,

\[ m = \rho \dot{v} = 1000 \times 40 = 40000 \text{ kg/s} \]

As per steady flow energy equation

\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q}_{1-2} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W}_{1-2} \text{ (no heat loss)} \]

To find Net energy

\[ m \left( u_1 + p_1 v_1 + \frac{C_1^2}{2} + Z_1 g \right) = m \left( u_2 + p_2 v_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W}_{1-2} \]

Net energy transfer from the fluid

\[ \dot{W}_{1-2} + \left( u_2 - u_1 \right) = m \left( P_1 v_1 - P_2 v_2 + \frac{C_1^2 - C_2^2}{2} + (Z_1 - Z_2) g \right) \]

\[ = 40000 \left[ (1.15 \times 10^6 \times 40 - 0.05 \times 10^6 \times 40) + \frac{30^2 - 15.5^2}{2} + (10 - 2) \times 9.81 \right] \]

\[ = 1.76 \times 10^{12} \text{ J/s} = 1.76 \times 10^6 \text{ MW} \]
**Problem 1.68:** In a Gas turbine installation, the gases enter the turbine at the rate of 5 kg/sec with a velocity of 50 m/sec and enthalpy of 900 kJ/kg and leave the turbine with 150 m/sec and enthalpy of 400 kJ/kg. The loss of heat from the gases to the surroundings is 25 kJ/kg. Assume $R = 0.285 \text{ kJ/kgK}$ $C_p = 1.004 \text{ kJ/kg K}$ and inlet conditions to be at 100 kPa and 27°C. Determine the diameter of the inlet pipe.

**Given** $m = 5 \text{ kg/s}$, $C_1 = 50 \text{ m/s}$, $h_1 = 900 \text{ kJ/kg}$

$C_2 = 150 \text{ m/s}$, $h_2 = 400 \text{ kJ/kg}$

$R = 0.285 \text{ kJ/kgK}$ $C_p = 1.004 \text{ kJ/kg K}$ $Q = -25 \text{ kJ/kg}$ $P_1 = 100 \text{ kPa; } t_1 = 27^\circ \text{C}$

The steady flow equation is given by

$$
\left(h_1 + \frac{C_1^2}{2} + Z_1 g\right) + Q = \left(h_2 + \frac{C_2^2}{2} + Z_2 g\right) + W
$$

Assume $Z_1 = Z_2$.

$$
\left(900 + \frac{(50)^2}{2 \times 1000}\right) - 25 = \left(400 + \frac{(150)^2}{2 \times 1000}\right) + W
$$

$$
876.25 = 411.25 + W
$$

$W = 465 \text{ kJ/kg}$

$W = m \dot{W} = 5 \times 465 = 2325 \text{ kW}$

To find $d_1$, mass flow rate $\dot{m} = \frac{A_1 C_1}{V_1}$

we know, $P_1 V_1 = mRT_1$

$$V_1 = \frac{RT_1}{P_1} = \frac{0.285 \times (27 + 273)}{100} = 0.855 \text{ m}^3$$

$$\dot{m} = \rho_1 A_1 C_1 = \frac{A_1 C_1}{V_1}$$

$$\therefore A_1 = \frac{\dot{m} V_1}{C_1} = \frac{5 \times 0.855}{50}$$

$A_1 = 0.0855 \text{ m}^2$
Problem 1.69: In a gas turbine unit, the gas flow rate is 15 kg/s and the power developed is 12000 kW. The enthalpy at inlet and exit are 1260 kJ/kg and 950 kJ/kg, while the velocities are 50 and 110 m/s respectively. Calculate the rate at which the heat is rejected and area of inlet pipe if specific volume at inlet is 0.45 m³/kg. (Nov/Dec 2009 - AU)

Solution: Given data:
\[ m = 15 \text{ kg/s}; \quad \text{Power } \dot{W} = 12000 \text{ kW}; \quad h_1 = 1260 \text{ kJ/kg}; \]
\[ h_2 = 950 \text{ kJ/kg}; \quad C_1 = 50 \text{ m/s}; \quad C_2 = 110 \text{ m/s}; \quad v_1 = 0.45 \text{ m}^3/\text{kg} \]

To find \( Q \)

The SFEE is
\[ \dot{Q} = \dot{m} \left[ \frac{C_2^2 - C_1^2}{2} + (h_2 - h_1) \right] + \dot{W} \]
\[ = 15 \left[ \frac{110^2 - 50^2}{2} + (950 - 1260) \times 10^3 \right] + 12000 \times 10^3 \]
\[ = 7422000 \text{ Watts} \]
\[ Q = 7422 \text{ kW} \]

To find \( A_1 \)

mass flow rate
\[ \dot{m} = \frac{A_1 C_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} v_1}{C_1} = \frac{15 \times 0.45}{50} = 0.135 \text{ m}^2 \]
1.53 COMPRESSOR

Compressor is a device used to increase the pressure of the fluid by taking work input.

**Problem 1.70:** Air enters a compressor with a velocity of 60 m/sec, pressure 100kPa, temp 40°C and leaves the compressor with a velocity of 90 m/sec, 500kPa and 120°C. Consider the system is adiabatic. Find the power of motor for the mass flow rate of 40kg/min. Write the assumptions made. (Nov/Dec 2006 - AU)

**Solution**

**Given**

\[ \begin{align*}
C_1 &= 60 \text{ m/sec} & C_2 &= 90 \text{ m/sec} \\
P_1 &= 100 \times 10^3 \text{ Pa} & P_2 &= 500 \times 10^3 \text{ Pa} \\
T_1 &= 40 + 273 = 313 \text{ K} & T_2 &= 120 + 273 = 393 \text{ K} \\
Q &= 0 \text{ (adiabatic)} \\
\dot{m} &= 40 \text{ kg/min} = \frac{40}{60} = 0.6667 \text{ kg/sec}
\end{align*} \]

**S.F.E.E**

\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W} \]

Assume \( Z_1 g = Z_2 g \)

(Since nothing is given about P.E.)

\[ \dot{W} = \dot{m} \left[ (h_1 - h_2) + \frac{C_1^2 - C_2^2}{2} \right] \]

\[ (h_1 - h_2) = C_p (T_1 - T_2) = 1.005 (313 - 393) \]

\[ = -80.4 \text{ kJ/kg} = -80.4 \times 10^3 \text{ J/kg} \]

\[ \dot{W} = 0.6667 \left[ (-80.4 \times 10^3) + \frac{60^2 - 90^2}{2} \right] \]

\[ = -55102.76 \text{ J/sec} = -55.103 \text{ kW} \]

[- sign indicates work is done on the system]
Problem 1.71: Air enters the compressor of a gas-turbine plant at ambient conditions of 100 kPa and 25°C with a low velocity and exists at 1 MPa and 347°C with a velocity of 90 m/s. The compressor is cooled at a rate of 1500 kJ/min, and the power input to the compressor is 250 kW. Determine the mass flow rate of air through the compressor. Assume \( C_p = 1.005 \text{ kJ/kg K}. \)  

(Nov/Dec 2016 - AU)

**Solution:**

\[ P_1 = 100 \text{ kPa} \]

\[ T_1 = 25 + 273 = 298 \text{ K} \]

\[ C_1 = 0; \ P_2 = 1000 \text{ kPa} \]

\[ T_2 = 347 + 273 = 620 \text{ K}; \ C_2 = 90 \text{ m/s}; \ C_p = 1.005 \text{ kJ/kg K} \]

\[ \dot{Q} = -1500 \text{ kJ/min} = -25 \text{ kJ/s} = -25 \text{ kW} \]

\[ \dot{W} = -250 \text{ kW} \]

Steady flow Energy Equation for compressor is

\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W} \]

\[ C_1 = 0; Z_1 = Z_2 \]

\[ \dot{m} \left( h_1 - h_2 - \frac{C_2^2}{2} \right) = \dot{W} - \dot{Q} \]

\[ \dot{m} \left( C_p \left( T_1 - T_2 \right) - \frac{C_2^2}{2} \right) = \dot{W} - \dot{Q} \]

\[ \dot{m} = \frac{-250 - (-25)}{[1.005 \left( 298 - 620 \right)] - \left( \frac{90^2}{2} \right)} = \frac{-225}{323.61 - 4050} = \frac{-225}{-4373.61} = 0.0514 \text{ kg/s} \]

Mass flow rate of air \( \dot{m} = 0.0514 \text{ kg/s} \)
Problem 1.72: Calculate the power required by a compressor if air flowing at the rate of 0.9 kg/s enters at 1.01 bar, 25°C with a velocity of 70 m/s and leaves at 2.02 bar, 75°C with a velocity of 140 m/s. Heat transferred from the air to the cooling water circulating around the compressor casing amounts to 16 kJ/kg of air.

Solution:

Given: 

\[ P_1 = 1.01 \text{ bar}; \quad P_2 = 2.02 \text{ bar}; \quad T_1 = 25^\circ \text{C}; \quad T_2 = 75^\circ \text{C} \]

\[ m = 0.9 \text{ kg/s} \quad C_1 = 70 \text{ m/s} \quad C_2 = 140 \text{ m/s} \]

by S.F.E.E.

\[
\begin{align*}
&\dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W} \\
&\dot{W} = -\dot{m} \left[ (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + (Z_2 - Z_1) g \right] + \dot{Q} \\
&= -0.9 \left[ 1.005 (75 - 25) + \frac{(140^2 - 70^2)}{2} \times 10^3 \right] - 16 \\
&= -63.89 \text{ kW}
\end{align*}
\]

Problem 1.73: Air flows steadily at the rate of 0.5 kg/s through an air compressor entering at 10 m/s velocity, 120 kPa pressure, and 0.90 m³/kg volume and leaving at 8 m/s, 740 kPa and 0.2 m³/kg. The internal energy of the air leaving is 95 kJ/kg greater than that of the air entering. Cooling water in the compressor jackets absorbs heat from the air at the rate of 60 kW.

(a) Compute the rate of shaft work input to the air in kW. (b) Find the ratio of the inlet pipe diameter to outlet pipe diameter. (Apr/May 2009 - AU)

Solution:

Given data:

\[ m = 0.5 \text{ kg/s} \quad C_1 = 10 \text{ m/s} \quad P_1 = 120 \text{ kN/m}^2 \quad v_1 = 0.90 \text{ m}^3/\text{kg} \]

\[ C_2 = 8 \text{ m/s} \quad P_2 = 740 \text{ kN/m}^2 \quad v_2 = 0.2 \text{ m}^3/\text{kg} \quad u_2 - u_1 = 95 \text{ kJ/kg} \quad Q = -60 \text{ kW} \]

\[ \text{[Note: Heat is flowing from air to Cooling water]} \]

To find work \( \dot{W} \) in kW

S.F.E.E is

\[
\dot{m} \left[ \frac{C_1^2}{2} + u_1 + P_1 v_1 \right] + \dot{Q} = \dot{m} \left[ \frac{C_2^2}{2} + u_2 + P_2 v_2 \right] + \dot{W} \]

\[ \text{[Note:} h = u + P v \text{]} \]
\[ \dot{W} = m \left[ \frac{C_2^2 - C_1^2}{2} + (u_1 - u_2) + (P_1 v_1 - P_2 v_2) \right] + \dot{Q} \]

\[ = 0.5 \left[ \frac{10^2 - 8^2}{2} + (-95 \times 10^3) + (120 \times 10^3 \times 0.9 - 740 \times 10^3 \times 0.2) \right] - 60 \times 10^3 \]

\[= 0.5 \left[ 18 - 95 \times 10^3 - 40,000 \right] - 60 \times 10^3 \]

\[ \dot{W} = -127,491 \text{ J/s} = -127.491 \text{ kW} \]

Rate of power input to air \( \dot{W} = -127.491 \text{ kW} \)

[- sign indicates work is done on the system]

To find \( \frac{d_1}{d_2} \)

Mass flow equation \( \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2} \)

\[ \frac{A_1}{A_2} = \frac{v_1}{v_2} \times \frac{C_2}{C_1} = \frac{0.9}{0.2} \times \frac{8}{10} = 3.6 \]

\[ \frac{A_1}{A_2} = \frac{\pi}{4} \frac{d_1^2}{d_2^2} = \frac{d_1^2}{d_2^2} = 3.6 \]

\[ \frac{d_1}{d_2} = \sqrt{3.6} = 1.89 \]

**Problem 1.74:** Air flows steadily at the rate of 0.4 kg/s through an air compressor, entering at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m³/kg and leaving at 4.5 m/s with a pressure of 6.9 bar and a specific volume of 0.16 m³/kg. The internal energy of air leaving is 88 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 59 W. Calculate the power required to drive the compressor and the inlet and outlet cross-sectional area.

(May/June 2012 - AU)

**Solution:** Given:

\[ \dot{m} = 0.4 \text{ kg/s} ; \ C_1 = 6 \text{ m/s} ; \ P_1 = 1 \text{ bar} = 100 \text{ kN/m}^2 ; \ v_1 = 0.85 \text{ m}^3/\text{kg} \]

\[ C_2 = 4.5 \text{ m/s} ; \ P_2 = 6.9 \text{ bar} = 6.9 \times 100 \text{ kN/m}^2 = 690 \text{ kN/m}^2 ; v_2 = 0.16 \text{ m}^3/\text{kg} \]

\[ u_2 - u_1 = 88 \text{ kJ/kg} ; \ Q = 59 \text{ W} \]
Solution

SFEE,

$$\dot{m}\left(u_1 + P_1 v_1 + \frac{C_1^2}{2} + Z_1 g\right) + \dot{Q} = \dot{m}\left(u_2 + P_2 v_2 + \frac{C_2^2}{2} + Z_2 g\right) + \dot{W}$$

$$\dot{m}\left(P_1 v_1 + \frac{C_1^2}{2} + Z_1 g\right) + \dot{Q} = \dot{m}\left([u_2 - u_1] + P_2 v_2 + \frac{C_2^2}{2} + Z_2 g\right) + \dot{W}$$

Assume, \( Z_1 = Z_2 \)

$$\Rightarrow 0.4 \left[(100 \times 0.85) + \frac{6^2}{2000}\right] + \frac{59}{1000} = 0.4 \left[88 + (690 \times 0.16) + \frac{4.5^2}{2000}\right] + \dot{W}$$

$$0.4 [85 + 0.018] + \frac{59}{1000} = 0.4 [88 + (110.4) + 0.010] + \dot{W}$$

$$34.066 = 79.36 + \dot{W}$$

$$\dot{W} = -45.29 \text{ kW}$$

[\text{\textquotesingle} \text{\textquotesingle} \text{ sign indicates work done on the system}]

Power required to drive the compressor \( \dot{W} = 45.29 \text{ kW} \)

To find \( A_1 \), \( \dot{m} = \frac{A_1 C_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} v_1}{C_1} = \frac{0.4 \times 0.85}{6} = 0.0567 \text{ m}^2 \)

To find \( A_2 \), \( \dot{m} = \frac{A_2 C_2}{v_2} \Rightarrow A_2 = \frac{\dot{m} v_2}{C_2} = \frac{0.4 \times 0.16}{4.5} = 0.01422 \text{ m}^2 \)

**Problem 1.75:** Air flows steadily at the rate of 0.5 kg/sec through an air compressor, entering at 7 m/sec with a pressure of 1.03 bar and a specific volume of 0.85 m$^3$/kg and leaving at 5.1 m/sec with a pressure of 7.5 bar and specific volume of 0.16 m$^3$/kg. The internal energy of leaving air is 90 kJ/kg higher than that of air at inlet. Heat rejected to the cooling water is 60 kJ/kg. Estimate the power required to run the compressor.  

*(Nov/Dec 2007 - AU)*

**Solution:**

**Given data:** \( m = 0.5 \text{ kg/sec} ; \quad C_1 = 7 \text{ m/sec} ; \)

\( P_1 = 1.03 \text{ bar} = 103 \text{ kN/m}^2 ; \quad v_1 = 0.85 \text{ m}^3/\text{kg} \)

\( C_2 = 5.1 \text{ m/sec} ; \quad P_2 = 7.5 \text{ bar} = 750 \text{ kN/m}^2 ; \)

\( v_2 = 0.16 \text{ m}^3/\text{kg} ; \quad u_2 - u_1 = \Delta u = 90 \text{ kJ/kg} ; \quad Q = -60 \text{ kJ/kg} \)
To find: Power required to run the Compressor ($\dot{W}$)

The SFEE for this system

$$\frac{C_1^2}{2} + h_1 + Q = \frac{C_2^2}{2} + h_2 + W \quad [\text{assuming } Z_1 = Z_2]$$

$$\frac{C_1^2}{2} + u_1 + P_1 v_1 + Q = \frac{C_2^2}{2} + u_2 + P_2 v_2 + W \quad [\because h = u + P v]$$

$$W = \frac{C_1^2 - C_2^2}{2} + (u_1 - u_2) + (P_1 v_1 - P_2 v_2) + Q$$

$$= \frac{7^2 - 5.1^2}{2} + (-90 \times 10^3) + (103 \times 10^3 \times 0.85 - 750 \times 10^3 \times 0.16) - 60 \times 10^3$$

$$= 11.495 - 90 \times 10^3 + (-32450) - 60 \times 10^3 = -18243.5$$

$$W = -18243.5 \text{ J/kg} = -182.4 \text{ kJ/kg}$$

Power $P = W \times \dot{m} = -182.4 \times 0.5 = -91.2 \text{ kJ/sec}$

Power required to drive the compressor $P = -91.2 \text{ kW}$

(-ve sign indicates that work is done on the system)

**Problem 1.76:** Air is compressed from 100 kPa and 22° C to a pressure of 1 MPa while being cooled at the rate of 16 kJ/kg by circulating water through the compressor casing. The volume flow rate of air at inlet condition is 150 m$^3$/min and power input to compressor is 500 kW. Neglecting the gravitational potential energy, determine the mass flow rate and the temperature of air at exit. (May/June 2012 - AU)

**Given data:**

$P_1 = 100 \text{ kPa} \; ; \; T_1 = 22° \text{ C} = 273 = 295 \text{ K} \; ; \; P_2 = 1 \text{ MPa} \; ; \; Q = -16 \text{ kJ/kg}$

$V_1 = 150 \text{ m}^3$/min \; ; \; $\dot{W} = 500 \text{ kW}$

**Solution**

$$\dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W}$$

...(1)

From Ideal gas equation:

$$P_1 \dot{V}_1 = \dot{m} R T_1$$
\[ m = \frac{100 \times 150}{0.287 \times 295} = 177.17 \text{ kg/min} = 2.952 \text{ kg/sec} \]

Neglecting the gravitational potential energy,

\( (Z_1 = Z_2 = 0) \)

Assume change in velocity head, \( \frac{C_1^2 - C_2^2}{2} = 0 \)

\[ \therefore \text{ The equation (1) is reduced to} \]

\[ m (h_1) + \dot{Q} = m (h_2) + \dot{W} \]

Assume, \( C_p = 1.005 \text{ kJ/kg K for air} \)

\[ \dot{m} (C_p T_1 + Q) = \dot{m} C_p T_2 + \dot{W} \]

\[ 2.952 \times (1.005 \times 295 - 16) = (2.952 \times 1.005 T_2) + 500 \]

\[ T_2 = 110.6 \text{ K} \]

**Result**

1. Mass flow rate, \( m = 2.952 \text{ kg/sec} \)
2. Temperature of air exit \( T_2 = 110.6 \text{ K} \)

**Problem 1.77:** An air compressor has a mass flow rate of air 100 kg/s at an inlet temperature and pressure of 15°C and 100 kN/m². Outlet conditions are 100°C and 180 kN/m². There is a cooling of 4 kW at the outlet surfaces of the compressor. Find the power required to drive the compressor. Assume that there is no change in velocity between inlet and outlet. (Apr/May 2007 - AU)

**Solution:**

Given data:

\[ \dot{m} = 100 \text{ kg/s} ; \quad T_1 = 15^\circ \text{ C} + 273 = 288 \text{ K} ; \quad P_1 = 100 \text{ kN/m}^2 \]

\[ T_2 = 100^\circ \text{ C} + 273 = 373 \text{ K} ; \quad P_2 = 180 \text{ kN/m}^2 \]

\[ \dot{Q} = -4 \text{ kW} = -4 \times 10^3 \text{ J/sec} \]

[\because \text{ Heat is transferred from air to cooling water}] \[ C_1 = C_2 \text{ and } Z_1 = Z_2 \]

To find Power \( (\dot{W}) \)

From equation of state, \( P_1 \dot{v}_1 = \dot{m}RT_1 \)

\[ \dot{v}_1 = \frac{\dot{m}RT_1}{P_1} = \frac{100 \times 0.287 \times 288}{100} = 82.656 \]

\[ \dot{v}_1 = 82.656 \text{ m}^3/\text{sec} \]
\[
\dot{v}_2 = \frac{\dot{m}RT_2}{P_2} = \frac{100 \times 0.287 \times 373}{180} = 59.47 \\
\dot{v}_2 = 59.47 \text{ m}^3/\text{sec}
\]

Change in internal energy \( \Delta u = u_2 - u_1 = \dot{m} C_v (T_2 - T_1) \)

\( = 100 \times 0.718 \times (373 - 288) = 6103 \)

\( u_2 - u_1 = 6103 \text{ kW} \)

SFEE for air compressor, \( (\dot{u}_1 + P_1 \dot{v}_1) + \dot{Q} = (\dot{u}_2 + P_2 \dot{v}_2) + \dot{W} \)

\[ \therefore \dot{W} = -6103 \times 10^3 + (100 \times 10^3 \times 82.656 - 180 \times 10^3 \times 59.47) - 4 \times 10^3 \\
= 8546000 \text{ J/sec} = 8546 \text{ kJ/sec} = -8546 \text{ kW} \]

Power required to drive the compressor \( \dot{W} = -8546 \text{ kW} \)

1.54 HEAT EXCHANGE

It is a device used to heat the fluid (or) cool the fluid. No work transfer, change in kinetic energy is negligible. Eg. Boiler cooling plant.

(i) Boiler:

It is a device in which water (or) fluid is heated.

Problem 1.78: The boiler generates 800 kg/hr of steam. The enthalpy of feed water is 170 kJ/kg and the enthalpy of steam is 2800 kJ/kg. Neglecting potential and kinetic energies, find the rate of heat transfer.

Solution

\[ \dot{m} = \frac{800}{3600} = 0.222 \text{ kg/sec} \]

S.F.E.E on time basis

\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 \right) + \dot{W} \]

Change in K.E. = 0; Change in P.E. = 0

\[ \dot{W} = 0 \]

The S.F.E.E equation becomes \( \dot{m} \left( h_1 \right) + \dot{Q} = \dot{m} \left( h_2 \right) \)
\[ Q = \dot{m} (h_2 - h_1) = 0.222 \times (2800 \times 10^3 - 170 \times 10^3) \]
\[ = 584444.4 \text{ J/sec} = 584.44 \text{ kW} \]

58.444 kW of heat is added to boiler water.

**Problem 1.79:** In a cooling plant water enters with enthalpy of 240 kJ/kg and leaves with enthalpy of 192 kJ/kg. Pump work is negligible. The exit pipe is 20 m above the inlet pipe. Change in K.E. is negligible. Find the heat transfer from the water.

### Solution

S.F.E.E. on mass basis

\[ h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \]

\[ W = 0 \]

\[ \frac{C_1^2}{2} = \frac{C_2^2}{2} ; Z_2 - Z_1 = 20 \text{ m} \]

The S.F.E.E. equation becomes

\[ h_1 + Z_1 g + Q = h_2 + Z_2 g \]

\[ Q = (h_2 - h_1) + (Z_2 g - Z_1 g) = (192 \times 10^3 - 240 \times 10^3) + (20) 9.81 \]

\[ Q = -47804.8 \text{ J/kg} = -47.803 \text{ kJ/kg} \]

(- sign indicates heat is flowing from the water to surroundings)

**Problem 1.80:** Write the steady flow equation as applied to a steam generator and an air compressor. In a gas turbine unit, a power of 1200 kW is developed for a gas flow rate of 15 kg/sec. The enthalpy and velocity of gases at the inlet and exit of the turbine are 1200 kJ/kg, 50 m/sec, 400 kJ/kg and 100 m/sec respectively. Estimate the rate of heat rejection and the area of the inlet pipe, if the specific volume of gases at the inlet is 0.45 m³/kg.

### Solution: Given data:

\[ \dot{W} = \text{Power} = 1200 \text{ kW} = 1200 \times 10^3 \text{ W} = 1200 \times 10^3 \text{ J/sec} \]

Mass flow rate \( \dot{m} = 15 \text{ kg/sec} \); \( h_1 = 1200 \text{ kJ/kg} \); \( C_1 = 50 \text{ m/s} \)

\( h_2 = 400 \text{ kJ/kg} \); \( C_2 = 100 \text{ m/sec} \); \( v_1 = 0.45 \text{ m}^3/\text{kg} \)

The SFEE for steam generator (boiler) \( Q = h_2 - h_1 \)

The SFEE for air compressor \( W = Q + (h_2 - h_1) \)
To find Heat rejection \( (Q) \),

SFEE for turbine
\[
\dot{m} \left[ \frac{C_1^2}{2} + h_1 \right] + \dot{Q} = \dot{m} \left[ \frac{C_2^2}{2} + h_2 \right] + \dot{W} \quad \text{[} \cdot \cdot Z_1 = Z_2 \text{]}
\]
\[
\dot{Q} = \dot{m} \left[ \frac{C_2^2 - C_1^2}{2} + (h_2 - h_1) \right] + \dot{W}
\]
\[
\dot{Q} = 15 \times \left[ \frac{100^2 - 50^2}{2} + (400 - 1200) \times 10^3 \right] + 1200 \times 10^3
\]
\[
= -10743750 \frac{J}{\text{sec}} = -10743.75 \text{ kW}
\]

(- sign indicates heat is lost to surroundings)

To find \( A_1 \)

Mass flow rate \( \dot{m} = \frac{A_1 C_1}{v_1} \)
\[
A_1 = \frac{\dot{m}v_1}{C_1} = \frac{15 \times 0.45}{50} = 0.135
\]
\[
A_1 = 0.135 \text{ m}^2
\]

**Problem 1.81:** Air at a temperature of 15°C passes through a heat exchanger at a velocity of 30 m/s where its temp. is raised to 800°C. It then enters a turbine with the same velocity of 30 m/s and expands until the temp. falls to 650°C. On leaving the turbine, the air is taken at a velocity of 60 m/s to a nozzle where it expands until the temperature has fallen to 500°C. If the air flow rate is 2 kg/s, Calculate,

(i) The rate of heat transfer to the air in the heat exchanger.
(ii) The power output from the turbine assuming no heat loss and
(iii) The velocity at exit from the nozzle, assuming no heat loss. Take the enthalpy of air as \( h = C_p t \), where \( C_p \) is the specific heat equal to 1.005 kJ/kg-K and \( t = \) the temperature.  

(Apr/May 2008 - AU)

**Solution**
\[
\dot{m} = 2 \text{ kg/s}; \quad h_1 = C_p T_1 = 1.005 \times 288 = 289.44 \frac{\text{kJ}}{\text{kg}}
\]
\[
h_2 = C_p T_2 = 1.005 \times 1073 = 1078.37 \frac{\text{kJ}}{\text{kg}}
\]
Steady Flow Energy Equation (S.F.E.E) for heat exchanger

\[ m \left( h_1 + Z_1g + \frac{C_1^2}{2} \right) + \dot{Q} = m \left( h_2 + Z_2g + \frac{C_2^2}{2} \right) + W \]

\( W = 0 \); \( Z_1 = Z_2 \); \( C_1 = C_2 \)

\[ m \dot{h}_1 + \dot{Q} = m \dot{h}_2 \]

\[ \dot{Q} = m (h_2 - h_1) \]

\[ = 2 \left( 1078.37 - 289.44 \right) \]

\[ \dot{Q} = 1577.85 \text{ kW} \]

**S.F.E.E for Turbine:**

\[ m \left( h_2 + Z_2g + \frac{C_2^2}{2} \right) + \dot{Q} = m \left( h_3 + \frac{C_3^2}{2} + Z_3g \right) + W \]

\( Z_2 = Z_3 \) No heat loss \( \rightarrow \) So \( \dot{Q} = 0 \)

\[ m \left( h_2 + \frac{C_2^2}{2} \right) = m \left( h_3 + \frac{C_3^2}{2} \right) + W \]

\[ \dot{W} = m \left[ (h_2 - h_3) + \left( \frac{C_2^2 - C_3^2}{2} \right) \right] \]

\[ h_3 = C_p (T_3) = 1.005 \times 923 = 927.62 \text{ kJ/kg} \]

\[ W = 2 \left[ \left( 1078.37 \times 10^3 - 927.62 \times 10^3 \right) + \left( \frac{30^2 - 60^2}{2} \right) \right] \]

\[ = 298800 \text{ watts} \]

\[ \dot{W} = 298.8 \text{ kW} \]

**Power output from the turbine = 298.8 kW**

**S.F.E.E for Nozzle:**

No heat loss \( \rightarrow \dot{Q} = 0 \); \( Z_1 = Z_2 \), \( \dot{W} = 0 \)

\[ m \left( h_3 + \frac{C_3^2}{2} \right) = m \left( h_4 + \frac{C_4^2}{2} \right) \]

\[ h_4 = C_p T_4 = 1.005 \times 773 = 76.87 \text{ kJ/kg} \]
Problem 1.82: In a gas turbine installation, air is heated inside heat exchanger up to 750°C from ambient temperature of 27°C. Hot air then enters into gas turbine with the velocity of 50 m/s and leaves at 600°C. Air leaving turbine enters a nozzle at 60 m/s velocity and leaves nozzle at temperature of 500°C. For unit mass flow rate of air, determine the following assuming adiabatic expansion in turbine and nozzle.

(i) Heat transfer to air in heat exchanger
(ii) Power output from turbine
(iii) Velocity at exit of nozzle

Take up for air as 1.005 kJ/kg K.  
(Apr/May 2011 - AU)

Given

\[ T_1 = 27^\circ \text{C} + 273 = 300 \text{ K}; \quad T_2 = 750^\circ \text{C} + 273 = 1023 \text{ K} \]
\[ C_1 = 50 \text{ m/s}; \quad C_2 = 50 \text{ m/s}; \]
\[ T_3 = 600^\circ \text{C} + 273 = 873 \text{ K}; \quad C_3 = 60 \text{ m/s}; \]
\[ T_4 = 500^\circ \text{C} + 273 = 773 \text{ K}; \quad \dot{m} = 1 \text{ kg/s} \]

\[
\frac{C_4^2}{2} = h_3 - h_4 + \frac{C_3^2}{2} \Rightarrow C_4^2 = 2 (h_3 - h_4) + C_3^2
\]
\[ C_4 = \sqrt{2 (h_3 - h_4) + C_3^2}
\]
\[ = \sqrt{2(927.62 \times 10^3 - 776.87 \times 10^3)} + 60^2 = 552.36 \text{ m/s}
\]

Exit velocity from nozzle = **552.36 m/s**
To find $Q_{1-2}, W_T, C_4$

Solution

From SFEE for heat exchanger and eliminating the terms not relevant

$$\left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + Q_{1-2} = \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + W_{1-2}$$

$$h_1 + Q_{1-2} = h_2$$

$\therefore Q_{1-2} = (h_2 - h_1)$

$= C_p (T_2 - T_1)$

$\therefore Q_{1-2} = 1.005 (1023 - 300)$

$$Q_{1-2} = 726.615 \text{ kJ/kg}$$

Energy equation for the turbine gives

$$\left( \frac{C_2^2}{2} + h_2 \right) = h_3 + \frac{C_3^2}{2} + W_T$$

$$\left( \frac{C_2^2 - C_3^2}{2} \right) + (h_2 - h_3) = W_T$$

$\therefore W_T = \left( \frac{50^2 - 60^2}{2 \times 1000} \right) + 1.005 (750 - 600)$

[Here K.E $\frac{C_2^2 - C_3^2}{2}$ is in J/kg. To convert to kJ/kg it is divided by 1000]

$$W_T = 150.74 \text{ kJ/kg}$$

Writing the energy equation for the nozzle

$$\frac{C_3^2}{2} + h_3 = \frac{C_4^2}{2} + h_4$$

$$\frac{C_4^2 - C_3^2}{2} = C_p (t_3 - t_4)$$

$C_4^2 - C_3^2 = 1.005 (600 - 500) \times 10^3 \times 2 = 201000$

$\therefore C_4^2 = 201000 + 60^2$

$$C_4 = 452.33 \text{ m/s}$$
1.55 APPLICATION OF FIRST LAW TO THE TRANSIENT FLOW (OR) UNSTEADY FLOW PROCESSES

Many engineering processes involve changes within the control volume over time i.e., the properties of the system (pressure, temperature, enthalpy) changes with time. Such processes are called transient flow processes or unsteady (variable) flow processes.

Two major principles are used in the analysis of transient flow processes. They are

(i) Conservation of mass

The law of mass conservation states that, “mass can neither be created nor destroyed”.

This principle can also be expressed as,

“The net change (increase or decrease) in the total mass of the system during a process is equal to the difference between the total mass entering and total mass leaving the system during that process. That is

\[
\Delta m_{\text{system}} = m_{\text{in}} - m_{\text{out}}
\]

This relation is often called as mass balance,

For control volume this is expressed as \( m_1 - m_v = (m_2 - m_1) c_v \)

(ii) Conservation of energy

The law of energy conservation states that, “Energy can neither be created nor destroyed, but it can change terms”.

This principle can also be expressed as,

The net change (increase or decrease) in the total energy of the system during a process is equal to the difference in total energy entering the system and total energy leaving the system during the process. That is

\[
E_{\text{in}} - E_{\text{out}} = (\Delta E)_{\text{system}}
\]

This relation is often called as energy balance.
1.55.1 Energy analysis of variable flow processes

Transient (or) variable flow processes can be analysed by the control volume technique.

Consider a device through which a fluid is flowing, under non-steady state conditions.

Applying conservation of mass and energy principle to the control volume,

\[ m_{\text{in}} = m_{\text{out}} = (\Delta m)_{cv} \]

where \((\Delta m)_{cv}\) = mass of fluid within the control volume at any instant of time.

\[ m_{\text{in}} - m_{\text{out}} = (m_f - m_i)_{cv} \]  

...\(1\)

Also,

\[ E_{\text{in}} - E_{\text{out}} = (\Delta E)_{cv} \]

\[ E_{\text{in}} - E_{\text{out}} = (E_2 - E_1)_{cv} \]  

...\(2\)

From the first law of thermodynamics,

\[ Q - W + \sum m_{\text{in}} E_{\text{in}} - \sum m_{\text{out}} E_{\text{out}} = E_{cv} \]

Total energy crossing boundary as heat and work

\[ Q - W + \sum m_{\text{in}} \left( h + \frac{C_{\text{in}}^2}{2} + gZ_{\text{in}} \right) - \sum m_{\text{out}} \left( h + \frac{C_{\text{out}}^2}{2} + gZ_{\text{out}} \right) \]

\[ = (m_f E_f - m_i E_i) \]

Therefore,

\[ Q - W + \sum m_{\text{in}} \left( h + \frac{C_i^2}{2} + gZ_i \right) - \sum m_{\text{out}} \left( h + \frac{C_f^2}{2} + gZ_f \right) \]

\[ = m_f \left( u_f + \frac{C_f^2}{2} + gZ_f \right) - m_i \left( u_i + \frac{C_i^2}{2} + gZ_i \right) \]
The above equation is the general energy equation for unsteady state flow processes.

Some commonly encountered transient flow processes are
(i) The charging process (or) the filling process
(ii) The discharging process (or) the emptying process

1.56 TANK FILLING PROCESS

Consider a tank being filled by the fluid from the pipeline. The following assumptions are made during the filling process. i.e.,
(i) Kinetic and potential energy of the fluid is very small, and assumed to be zero.
(ii) Work involved in the opening of valve is negligible, \( W = 0 \)
(iii) Also, no heat transfer takes place across the system boundary, \( Q = 0 \)
(iv) Enthalpy of the system during filling remains uniform.

Then, the general energy equation reduces to,

\[
\sum m_i h_i - \sum m_o h_o = m_f u_f - m_i u_i \tag{3}
\]
Also for total filling process
Mass of fluid entering the tank = Final mass of the fluid in
\[ m_{in} = m_f \]

Mass of fluid leaving the tank = Initial mass of the fluid (suppose if the tank is drained before the actual filling takes place)
\[ m_{out} = m_i \]

then the above equation becomes,
\[ (m_f - m_i) \cdot h = m_f \cdot u_f - m_i \cdot u_i \] ...(4)

**Case (i):** Consider if the tank is empty, \( m_i = 0 \) initially
Then the above equation (4) becomes,
\[ m_f \cdot h = m_f \cdot u_f \]

Where \( h = \) Enthalpy of the fluid
\( m_f = \) Final mass of the fluid in the tank
\( u_f = \) Final internal energy of the fluid

On simplifying, the above equation becomes,
\[ h = u_f \]

Suppose, if the fluid used is an ideal gas, then the temperature of the gas in the tank after it is charged, becomes
\[ C_p \cdot T = C_v \cdot T_f \Rightarrow T_f = \frac{C_p}{C_v} \cdot T \]
\[ T_f = \gamma T \]

### 1.57 TANK EMPTYING PROCESS (TANK DISCHARGING PROCESS)

Consider a cylinder containing gas at higher pressure. If the valve is opened, the gas inside the cylinder rushes out into the surroundings. Now, the condition of the gas which is inside the cylinder varies with time.

![Fig. 1.51](image-url)
Assumptions

(i) Potential energy of the gas is small & assumed to be zero.
(ii) Work involved in the opening of valve is negligible, \( W = 0 \).
(iii) Velocity \( (C_d) \) and enthalpy \( (h_d) \) of the system during discharging remains uniform.

Applying, the above assumptions to the general energy equation for transient flow, the equation becomes,

\[
Q + \sum_{\text{in}} m \left( h_d + \frac{C_d^2}{2} \right) - \sum_{\text{out}} m \left( h_d + \frac{C_d^2}{2} \right) = m_f u_f - m_i u_i
\]

Also for total emptying process,

Mass of the fluid leaving the cylinder = Initial mass of the fluid in the cylinder

\( m_{\text{out}} = m_i \)

mass of the fluid, that is left out in the cylinder = Final mass of the fluid in the cylinder

\( m_{\text{in}} = m_f \)

Therefore, the equation of emptying process is further simplified as

\[
Q = (m_f u_f - m_i u_i) + (m_{\text{out}} - m_{\text{in}}) \left( h_d + \frac{C_d^2}{2} \right)
\]
1.58 SOLVED PROBLEMS ON UNSTEADY FLOW PROCESS:

Problem 1.83 An insulated rigid tank of 1.5 m³ of air with a pressure of 5 bar and 100°C discharges air into the atmosphere which is at 1 bar through a discharge pipe till its pressure becomes 1 bar. Calculate (a) the velocity of air in the discharge pipe (b) Evaluate the work that can be obtained through frictionless turbine using kinetic energy of that air.

Given Data:
Total volume \( V = 1.5 \text{ m}^3 \)
Initial pressure \( P_i = 5 \text{ bar} = 500 \text{ kPa} \)
Final pressure \( P_f = 1 \text{ bar} = 100 \text{ kPa} \)
Initial temperature \( T_i = 100°C = 373 \text{ K} \)

To find:
(a) Velocity \( (C_d) \)  (b) Turbine work \( (W) \)

Solution:
This problem is based on Tank discharging process.
\[ Q = (m_f u_f - m_i u_i) + (m_i - m_f) \left( h_d + \frac{C_d^2}{2} \right) \]

Since, the tank is insulated, \( Q = 0 \)

Consider it as an adiabatic process, \( \frac{T_f}{T_i} = \left( \frac{P_f}{P_i} \right)^{\frac{\gamma-1}{\gamma}} \)

\[ T_f = 373 \times \left( \frac{100}{500} \right)^{\frac{0.4}{1.4}} = 236 \text{ K} \]

\[ \Rightarrow m_i = \frac{P_i V_i}{RT_i} = \frac{500 \times 1.5}{0.287 \times 373} = 7 \text{ kg} \]

\[ m_f = \frac{P_f V_f}{RT_f} = \frac{100 \times 1.5}{0.287 \times 236} = 2.21 \text{ kg} \]

\[ 0 = [(2.21 \times 0.718 \times 236) - (7 \times 0.718 \times 373)] + (7 - 2.21) \left[ (1.005 \times 236) + \frac{C_d^2}{2000} \right] \]
\[ C_d = \sqrt{2000 \left[ \frac{(1874.69 - 374.48) - 237.18}{4.79} \right]} = 593.3 \text{ m/s} \]

(b) Turbine work \( W \) = kinetic energy = \( \frac{1}{2} m C_d^2 \)
\[ = \frac{1}{2} (4.79 \times 593.3^2) \]
\[ W = 843.05 \text{ kJ} \]

**Problem 1.84:** An insulated rigid CO\(_2\) cylinder of 3 m\(^3\) is initially at pressure of 16 bar and 42\(^\circ\)C. As the gas slowly leaks to atmosphere, its pressure finally becomes 12 bar. Find the mass of the gas leaked and change in internal energy of the gas. (Consider \( \gamma = 1.33 \) & \( R = 0.189 \text{ kJ/kgK} \))

**Given Data:**

\( P_i = 16 \text{ bar} = 1600 \text{ kPa}; \quad P_f = 12 \text{ bar} = 1200 \text{ kPa} \)
\( V = 3 \text{ m}^3; \quad T_i = (42 + 273) = 315 \text{ K} \)

**To find**

(a) Mass of gas leaked \( m \)  
(b) Change in internal energy \( \Delta V \)

**Solution:**

As it is insulated, it can be considered as adiabatic process

\[ \frac{T_f}{T_i} = \left( \frac{P_f}{P_i} \right)^{\frac{\gamma - 1}{\gamma}} \]
\[ T_f = 315 \times \left( \frac{1200}{1600} \right)^{0.33} = 280.33 \text{ K} \]

Initial mass \( m_i = \frac{P_i V_i}{RT_i} = \frac{1600 \times 3}{0.189 \times 315} = 80.644 \text{ kg} \)

Final mass \( m_f = \frac{P_f V_f}{RT_f} = \frac{1200 \times 3}{0.189 \times 280.33} = 67.94 \text{ kg} \)

\( \Rightarrow \) Mass of gas leaked \( m = m_i - m_f = 80.644 - 67.94 \)
\[ m = 12.704 \text{ kg} \]

Change in internal energy \( \Delta U = m_f u_f - m_i u_i \)
\[ \Delta U = m_f \cdot C_v \cdot T_f - m_i \cdot C_v \cdot T_i = C_v \cdot (m_f \cdot T_f - m_i \cdot T_i) \]

\[ \therefore C_v = \frac{R}{\gamma - 1} \]

\[ = 0.573 \left( 67.94 \times 280.33 - 80.64 \times 315 \right) \]

\[ \Delta U = -3641.97 \text{ kJ} \]

**Problem 1.85:** An insulated rigid tank of 1.45 m\(^3\) contains air at 3 bar at 303 K. It is connected to an air supply line with air at 10 bar and 323 K. The interconnecting valve is slowly opened and air supply line is allowed to enter the tank until the pressure in the tank reaches 10 bar. Find the mass of air added to the cylinder and the final temperature of air in cylinder.

**Given data:**

\[ V = 1.45 \text{ m}^3; \quad P_i = 3 \text{ bar} = 300 \text{ kPa}; \quad T_i = 303 \text{ K}; \quad P_s = 1000 \text{ kPa} \]

\[ T_s = 323 \text{ K}; \quad P_f = 1000 \text{ kPa} \]

**To find:**

(a) Mass of air added to cylinder \((m_a)\)
(b) Final temperature \((T_f)\)

**Solution:**

In case of Tank filling process \(Q = 0\) for insulated tank.

\[ m_i = \frac{P_i \cdot V_i}{R \cdot T_i} = \frac{300 \times 1.45}{0.287 \times 303} = 5.002 \text{ kg} \]

\[ \text{...}(i) \]

\[ m_f = \frac{P_f \cdot V_f}{R \cdot T_f} \]

\[ \text{...}(ii) \]

\[ \text{and} \quad m_f = \frac{P_f}{P_i} \cdot \frac{T_f}{T_i} \]

\[ m_i = \frac{P_i}{P_f} \cdot \frac{T_i}{T_f} \]

\[ m_i \left( m_f - m_i \right) = m_i \left( \frac{m_f}{m_i} - 1 \right) (h_s) \]

\[ \left[ \frac{P_f}{P_i} \right] \left[ \frac{T_f}{T_i} \right] C_v \cdot T_f - C_v \cdot T_i = \left[ \frac{P_f}{P_i} \right] \left[ \frac{T_f}{T_i} - 1 \right] (C_p \cdot T_s) \]

\[ \Rightarrow \left[ \frac{P_f}{P_i} \right] \left[ \frac{T_f}{T_i} - 1 \right] = \left[ \frac{P_f}{P_i} \right] \left[ \frac{T_f}{T_i} - 1 \right] (\gamma \cdot T_s) \quad (+ \text{ by } C_v) \]
Problem 1.86: An insulated rigid tank having 5 kg of air at 3 atm and 30°C is connected to an air supply line at 8 atm and 50°C through a valve. The valve is now slowly opened to allow the air from the supply line to flow into the tank until the tank pressure reaches 8 atm, and then the valve is closed. Determine the final temperature of the air in the tank. Also, find the amount of air added to the tank. (Nov/Dec 2015 - AU)

Solution

For insulated tank, \( Q = 0 \)

\[ W = 0 \]

The mass conservation equation

\[ m_s = (m_2 - m_1) \]

Conservation of energy

\[ m_s h_s = (m_2 u_2 - m_1 u_1) \ldots (1) \]

Equation (1) becomes

\[ (m_2 - m_1) C_p T_s = C_v (m_2 T_2 - m_1 T_1) \ldots (2) \]

\[ PV = mRT \]

\[ m_1 = \frac{P_1 V_1}{RT_1} \]

\[ m_2 = \frac{P_2 V_2}{RT_2} \]

\[ \frac{m_2}{m_1} = \frac{P_2}{P_1} \frac{T_1}{T_2} \quad [\therefore V_1 = V_2] \]
\[ \frac{m_2}{m_1} = \frac{8}{3} \left( \frac{T_1}{T_2} \right) \]

\[ m_2 = \frac{8}{3} \left( \frac{T_1}{T_2} \right) m_1 \]

subs. \( m_2 \) in equation (2)

\[
\left[ \frac{8}{3} \frac{T_1}{T_2} m_1 - m_1 \right] C_p T_s = C_v \left[ \frac{8}{3} \frac{T_1}{T_2} m_1 T_2 - m_1 T_1 \right] \\
\left( \frac{8}{3} \frac{T_1}{T_2} - 1 \right) C_p T_s = C_v \left( \frac{8}{3} \frac{T_1}{T_2} T_1 \right) \\
\left( \frac{8}{3} \frac{T_1}{T_2} - 1 \right) \gamma T_s = \left( \frac{8}{3} \frac{T_1}{T_2} \right) T_1 \\
\left( \frac{8}{3} \times \frac{303}{T_2} - 1 \right) 1.4 \times 323 = \left( \frac{8}{3} \frac{T_1}{T_2} \right) 303
\]

\[ \therefore \text{ Final temperature } T_2 = 381 \cdot 715 K \]

Also; \( \frac{C_p}{C_v} = \gamma; \quad \gamma = 1.4 \)
\[
\therefore \frac{m_2}{m_1} = \frac{8}{3} \left( \frac{303}{381.715} \right) \\
\frac{m_2}{m_1} = 2.12 \implies m_2 = 2.12 \times m_1 \\
\therefore m_2 = 2.12 \times 5 = 10.6 \, \text{kg} \\
\therefore \text{The mass of air added } \Delta m = m_3 = m_2 - m_1 = 10.6 - 5 = 5.6
\]

\[
\Delta m = m_3 = 5.6 \, \text{kg}
\]

**Problem 1.87:** A tank of volume 0.45 m\(^3\) contains an electric heating coil but it is thermally insulated. The tank is equipped with release valve which allows gas to escape when pressure in the tank reaches 8 bar. The tank is filled with O\(_2\) at a pressure of 3 bar and a temperature of 20°C and gas is then heated until the temperature in the tank is 1150°C find the heat transfer (a) up to the time when release value opens (b) after this time. Assume \(C_p = 1.0748 \, \text{kJ/kgK}; \ C_v = 0.748 \, \text{kJ/kgK}\)

**Given Data:**

\[
V = 0.45 \, \text{m}^3; \ P_i = 2 \, \text{bar} \ P_f = 8 \, \text{bar} \\
T_i = 20^\circ \text{C} = 20 + 273 = 293 \, \text{K} \ ; \ T_f = 1150^\circ \text{C} = 1150 + 273 = 1423 \, \text{K} \\
C_p = 1.0748 \, \text{kJ/kgK}; \ C_v = 0.748 \, \text{kJ/kgK}
\]

**To find:**

(a) Heat transfer up to the time when release valve opens.
(b) Heat transfer after release of valve.

**Solution:**

\[
m_i = \frac{P_i V_i}{RT_i} = \frac{3 \times 100 \times 0.45}{0.297 \times 293} = 1.551 \, \text{kg}
\]

The valve opens at constant volume process

\[
T_o = \frac{P_f}{P_i} \times T_i = 293 \times \frac{800}{300} = 781 \, \text{K}
\]

Heat transfer \(Q = mC_v (T_2 - T_i)\)

\[
= 1.551 \times 0.748 \times (781 - 293) = 566.15 \, \text{kJ}
\]
Heat transfer \( Q \) after release

\[
Q = C_p P_f V \ln \left( \frac{T_f}{T_o} \right)
\]

\[
= \frac{1.045 \times 800 \times 0.45}{0.297} \ln \left( \frac{1423}{781} \right)
\]

\( Q = 760 \text{ kJ} \)

**Problem 1.88:** An insulated and rigid vessel contains 16 m³ of air at 20 bar and 200°C. This allows to expand to 4 bar. Find the maximum work that can be obtained from escaping air in an adiabatic process. Take \( \gamma = 1.4 \) & \( R = 0.277 \text{ kJ/kgK} \). \( C_p = 1.005 \text{ kJ/kgK} \), \( C_v = 0.728 \text{ kJ/kgK} \)

**Given data:**

\( V = 16 \text{ m}^3 \); \( P_f = 20 \text{ bar} = 2000 \text{ kPa} \); \( T_i = 200°C = 473 \text{ K} \)

\( P_f = 4 \text{ bar} = 400 \text{ kPa} \); \( R = 0.277 \text{ kJ/kgK} \) \( C_p = 1.005 \text{ kJ/kgK} \); \( C_v = 0.728 \text{ kJ/kgK} \)

**To find:**

Maximum work

**Solution:**

\[
\frac{T_i}{T_f} = \left( \frac{P_i}{P_f} \right)^{\frac{\gamma - 1}{\gamma}} = \left( \frac{20}{4} \right)^{\frac{1.4}{1.4}} \Rightarrow T_f = 244.99 \text{ K}
\]

\[
m_i = \frac{P_i V_i}{R T_i} = \frac{2000 \times 16}{0.277 \times 473} = 244.23 \text{ kg}
\]

\[
m_f = \frac{P_f V_f}{R T_f} = \frac{400 \times 16}{0.277 (244.99)} = 94.3 \text{ kg}
\]

We know that,

\[
Q - W = \Delta E + (m_2) e_2 - (m_1) e_1
\]

\[
\therefore Q = 0 \text{ for insulated tank}
\]

\[
- W = (m_f u_f - m_i u_i) + m_2 \left( h_f + \frac{C_s^2}{2} \right) - m_1 \left( h_i - \frac{C_s^2}{2} \right)
\]

Here \( m_1 = 0 \), \( C_s = 0 \)

\[
- W = [(94.3 \times 0.728 \times 244.9) - (244.24 \times 0.728 \times 473)]
\]

\[
+ [(94.3 - 244.24) \times 1.005 \times 244.9]
\]

\[
- W = - 104194 \text{ kJ}
\]

\[
W = 104194 \text{ kJ}
\]
# Limitations of First Law of Thermodynamics

**List the limitations of First law of Thermodynamics (Nov/Dec 2012 - AU)**

<table>
<thead>
<tr>
<th>Limitations</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) No restriction in the direction of heat flow.</td>
<td>First law does not specify the direction of the process. It only establishes a relationship between heat and work, but failed to mention, whether the heat flows from cold body to hot body or not.</td>
<td>Consider heating of a room by means of an electric room heater. Room gets heated by passing current through a resistor. But, transferring the heat from room will not generate electrical energy.</td>
</tr>
<tr>
<td>(ii) Feasibility of the process is not specified.</td>
<td>It does not give any information, whether the process is possible or not.</td>
<td>Consider two bodies $A$ and $B$. $A$ is at higher temperature than $B$. When brought together, either the body $A$ loses heat to body $B$ or the body $A$ gains heat from body $B$. First law fails to explain the process that is possible.</td>
</tr>
<tr>
<td>(iii) Amount of energy conversion is not specified.</td>
<td>Work can be converted into 100% equivalent heat but heat cannot be converted into 100% equivalent work. So heat and work are not completely interchangeable forms of energy.</td>
<td>An electric heater can convert 1 kJ of work to 1 kJ heat but an IC engine cannot convert 1 kJ heat to 1kJ work. This is because some amount of heat get dissipated during combustion process.</td>
</tr>
</tbody>
</table>
IMPORTANT FORMULA AND HINTS

Unit 1: Basic Concepts and First Law of Thermodynamics

Units & Dimensions

(i) Mass (m) \(\rightarrow\) Unit is ‘kg’ in SI system. 
(ii) Force (F) \(\rightarrow\) Unit is ‘N’ in SI system. \(F = ma,\)

Where, \(m \rightarrow\) Mass
\(a \rightarrow\) Acceleration
(iii) Pressure (P) \(\rightarrow\) Unit is ‘Pa’ in SI system
\[ P = \lim_{{\delta A \to \delta A'}} \frac{\delta F_n}{\delta A} \]
Where, \(\delta A \rightarrow\) A small area
\(\delta A' \rightarrow\) The smallest area from continuum
\(\delta F_n \rightarrow\) Force component normal to \(\delta A\)

(iv) Temperature (T) \(\rightarrow\) Unit ‘K’ in SI system
(v) Specific volume (v) \(\rightarrow\) Unit ‘m³/kg’ in SI system
\[ v = \frac{V}{m} \]
Where \(V \rightarrow\) Total volume
\(m \rightarrow\) mass

(vi) Energy (E) \(\rightarrow\) ‘J’ in SI system
(vii) Power \(\rightarrow\) Unit ‘W’ in SI system

Thermodynamic system

Thermodynamic system is known as a space of constrained area upon which our attention is concentrated on.
The system is classified into:
(i) Closed system
(ii) Open system
(iii) Isolated system
Closed system

No mass transfer occurs across the boundary, but energy, in the form of heat or work can cross the boundary.

Open system

Both energy and mass can transfer from the system to surroundings and from the surroundings to system.
**Isolated system**

Neither energy nor mass transfer occurs between the system and surroundings.

**Property**

It is a characteristic of the system.

**Intensive property**

Intensive property is independent of mass of the system.

**Fig. 1.8**

Those properties which do not depend on the mass of the system. 

**Eg.** Temperature, Pressure, Specific volume, etc.
Extensive property
Those properties which depend on the mass of the system.
Eg. Total volume, Total energy, etc.

State
State is the condition of the system at an instant of time as described by its properties.

Process
A process occurs when the system undergoes a change in a state or when the energy transfers at a steady state.

A Quasi-static process is a succession of equilibrium states and infinite slowness is its characteristics feature.

Thermodynamic Equilibrium
When mechanical equilibrium, chemical equilibrium and thermal equilibrium are satisfied, then the system is said to be in thermodynamic equilibrium.

Different forms of Energy
(i) Potential Energy (PE) \( \rightarrow \) Unit ‘J’ in SI system
\[
P.E = mgZ
\]
Where \( m \rightarrow \) Mass
\( g \rightarrow \) Acceleration due to gravity
\( Z \rightarrow \) Height

(ii) Kinetic Energy (K.E) \( \rightarrow \) Unit ‘J’ in SI system
\[
K.E = \frac{1}{2} m V^2 = \frac{1}{2} mC^2
\]
Where, \( m \rightarrow \) Mass
\( C \) (or) \( V \rightarrow \) Velocity

(iii) Pressure Energy \( \rightarrow \) Unit ‘J/kg’ in SI system
(iv) Internal Energy \( (U) \rightarrow \) Unit ‘J’ in SI system
(v) Heat \( (Q) \rightarrow \) Unit ‘J’ in SI system

Note:
Heat flowing into system is considered as positive \(( + Q )\)
Heat flowing out of system is considered as negative \(( - Q )\)
(vi) Specific Heat \((C)\) → Unit ‘J/kgK’ in SI system

\[
C = \frac{Q}{m \Delta T}
\]

Where
- \(Q\) → Heat flow
- \(m\) → Mass
- \(\Delta T\) → Change in Temperature

**Note:**
For gases, \(C\) depends on the process, \(C_p\) for constant pressure process, \(C_v\) for constant volume process.

(vii) Enthalpy \((H)\) → Unit ‘J’ in SI system

\[
H = U + PV
\]

Where,
- \(U\) → Internal Energy
- \(P\) → Pressure
- \(V\) → Volume

(viii) Work \((W)\) → Unit ‘J’ in SI system

\[
W = F \times x,
\]

Where,
- \(F\) → Force
- \(x\) → Displacement in the direction of applied force

**Note:**
When the work is done by the system, it is positive work.
When the work is done on the system, it is negative work.
Different Modes of work

(i) **Electrical work** ($W_e$):

$$ W_e = VI \text{ in Watts} $$

Where,  
$I$ → Current  
$V$ → Potential difference

(ii) **Shaft work** ($W_{sh}$):

$$ W_{sh} = \frac{2\pi NT}{60} \text{ in Watts} $$

$$ T = F \times r $$

Where,  
$N$ → Number of revolution per minute  
$T$ → Torque  
$F$ → Force  
$r$ → Radius

(iii) **Spring work** ($W_{spring}$):

$$ \delta W_{spring} = F \cdot dx \text{ [kJ]} $$

Where,  
$F$ → Force  
$k$ → Spring constant  
$x$ → Spring displaced distance
(iv) Paddle wheel work (or) Stirring work \( (W_{\text{stir}}) \)

\[
W = \int_{1}^{2} mg \, dz = \int_{1}^{0} Td \, \theta
\]

(v) Flow Work \( (W_{\text{flow}}) \)

\[
W_{\text{flow}} = PV \quad [\text{kJ}]
\]

Where, \( P \rightarrow \) Pressure  \\
\( V \rightarrow \) Volume

(vi) Work transfer \( W_{1-2} = \int_{1}^{2} PdV \)

First Law of Thermodynamics
It states that energy can be neither created nor destroyed, it can only change forms.

For closed system, (In a cycle)
Net heat transfer = Net work transfer 

\[
\oint dQ = \oint dW
\]

For a process,

\[
Q = \Delta E + W,
\]
\[
E = KE + PE + U
\]

Where, \( Q \rightarrow \) Total heat flow  \\
\( \Delta E \rightarrow \) Change in energy  \\
\( W \rightarrow \) Work transfer
For all processes, \( \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \) and \( PV = mRT \)

<table>
<thead>
<tr>
<th>Process</th>
<th>( pV^n = C )</th>
<th>( W )</th>
<th>( \Delta U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Constant volume process</td>
<td>( n = \infty )</td>
<td>( W = \int pdv ) = 0 (zero)</td>
<td>( mc_v(T_2 - T_1) )</td>
</tr>
<tr>
<td></td>
<td>( V = C )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Constant pressure process</td>
<td>( n = 0 )</td>
<td>( W = P(V_2 - V_1) ) (or)</td>
<td>( mc_v(T_2 - T_1) )</td>
</tr>
<tr>
<td></td>
<td>( P = C )</td>
<td>( W = mR(T_2 - T_1) )</td>
<td></td>
</tr>
<tr>
<td>3. Constant temperature process (or) Isothermal process</td>
<td>( n = 1 ) ( PV = C ) ( T = C )</td>
<td>( PV \ln \left( \frac{V_2}{V_1} \right) ) or ( PV \ln \left( \frac{P_2}{P_1} \right) ) or ( mRT \ln \left( \frac{V_2}{V_1} \right) )</td>
<td>( 0 ) (zero) (since ( T_1 = T_2 ))</td>
</tr>
<tr>
<td>4. Isentropic process (or) Reversible Adiabatic process</td>
<td>( n = \gamma ) ( PV^n = C )</td>
<td>( \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} ) (or) ( mR(T_1 - T_2) ) ( \frac{n - 1}{\gamma - 1} )</td>
<td>( mc_v(T_2 - T_1) ) or ( \Delta U = -W )</td>
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<td></td>
<td>( T_2 = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} )</td>
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<td></td>
<td>( T_2 = \left( \frac{V_2}{V_1} \right)^{\gamma - 1} )</td>
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<tr>
<td>5. Polytropic process</td>
<td>( n = n; ; pV^n = C )</td>
<td>( \frac{P_1 V_1 - P_2 V_2}{n - 1} ) (or) ( mR(T_1 - T_2) ) ( \frac{n - 1}{n - 1} )</td>
<td>( mc_v(T_2 - T_1) )</td>
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<td></td>
<td>( T_2 = \left( \frac{P_2}{P_1} \right)^{\frac{n - 1}{n}} )</td>
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<tr>
<td></td>
<td>( V_2 = \left( \frac{P_1}{P_2} \right)^{\frac{1}{n}} )</td>
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</tbody>
</table>
\[ P_1 V_1 = mRT_1; \quad P_2 V_2 = mRT_2 \] ...

<table>
<thead>
<tr>
<th></th>
<th>( Q )</th>
<th>( \Delta H )</th>
<th>( \Delta S )</th>
<th>Remarks</th>
</tr>
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<tr>
<td>Heat transfer</td>
<td>Change in Enthalpy</td>
<td>Change in Entropy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>( mC_v(T_2 - T_1) )</td>
<td>( mC_v(T_2 - T_1) )</td>
<td>( mC_v \ln \left( \frac{T_2}{T_1} \right) ) (or) ( mC_v \ln \left( \frac{P_2}{P_1} \right) )</td>
<td>( \int PdV = 0 ) since ( dV = 0 ); ( Q = \Delta U; \quad V_1 = V_2 )</td>
</tr>
<tr>
<td>2.</td>
<td>( mC_p(T_2 - T_1) )</td>
<td>( mC_p(T_2 - T_1) )</td>
<td>( mC_p \ln \left( \frac{T_2}{T_1} \right) ) (or) ( mC_p \ln \left( \frac{V_2}{V_1} \right) )</td>
<td>( Q = \Delta H ); ( P_1 = P_2 )</td>
</tr>
<tr>
<td>3. ( Q = W ) ( = PV \ln \left( \frac{V_2}{V_1} \right) ) ( = PV \ln \left( \frac{P_1}{P_2} \right) )</td>
<td>0 (zero) ( ) (since ( T_1 = T_2 )).</td>
<td>( mR \ln \left( \frac{V_2}{V_1} \right) ) (or) ( mR \ln \left( \frac{P_1}{P_2} \right) )</td>
<td>( Q = W; \quad P_1 V_1 = P_2 V_2 ); ( T_1 = T_2 )</td>
<td></td>
</tr>
<tr>
<td>4. Zero ( ) (No Heat Transfer)</td>
<td>( mC_p(T_2 - T_1) )</td>
<td>0 (zero) since ( s_1 = s_2 )</td>
<td>( P_1 V_1^n = P_2 V_2^n ); ( T_2 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = \left( \frac{V_1}{V_2} \right)^{\frac{n-1}{n}} ); ( \frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{\frac{n-1}{n}} ); ( \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} )</td>
<td></td>
</tr>
<tr>
<td>5. ( \left[ \frac{\gamma - n}{\gamma - 1} \right] \times W ) ( ) (or) ( mC_p(T_2 - T_1) )</td>
<td>( mC_n \ln \left( \frac{T_2}{T_1} \right) ) (or) ( \frac{\gamma - n}{\gamma - 1} mR \ln \left( \frac{V_2}{V_1} \right) )</td>
<td>Polytropic sp. heat ( C_n ) ( )</td>
<td>( C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) ); ( P_1 V_1^n = P_2 V_2^n ); ( T_2 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = \left( \frac{V_1}{V_2} \right)^{\frac{n-1}{n}} ); ( \frac{T_2}{T_1} = \left( \frac{V_1}{V_2} \right)^{\frac{n-1}{n}} )</td>
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</tbody>
</table>
Application of First Law to Non-flow process (or) Closed system

(i) For constant volume process, (or) Isochoric process

\[ V = \text{constant} \quad (V_1 = V_2) \]

\[ n = \infty \]

Work done, \( W = 0 \)

Change in internal energy, \( \Delta U = m \ C_v (T_2 - T_1) \)

Heat transfer, \( Q = m \ C_p (T_2 - T_1) \)

Change in Enthalpy, \( \Delta H = m \ C_p (T_2 - T_1) \)

Change in Entropy, \( \Delta S = m \ C_v \ln \left( \frac{T_2}{T_1} \right) = m \ C_v \ln \left( \frac{P_2}{P_1} \right) \)

Note:

For const. volume process, \( \int PdV = 0 \)

\[ Q = \Delta U \]

(ii) For constant pressure process, (or) Isobaric process

\[ P = \text{constant} \quad (P_1 = P_2) \]

\[ n = 0 \]

Work done, \( W = P (V_2 - V_1) = mR (T_2 - T_1) \)

Change in internal energy, \( \Delta U = m \ C_v (T_2 - T_1) \)

Heat Transfer, \( Q = m \ C_p (T_2 - T_1) \)

Change in Enthalpy, \( \Delta H = m \ C_p (T_2 - T_1) \)

Change in Entropy, \( \Delta S = m \ C_p \ln \left( \frac{T_2}{T_1} \right) = m \ C_p \ln \left( \frac{V_2}{V_1} \right) \)

Note:

For const. pressure process, \( Q = \Delta H \)

(iii) Constant temperature process, (or) Isothermal process

\[ n = 1 \]

\[ T = \text{const} \quad (T_1 = T_2) \]
Workdone, \( W = PV \ln \left( \frac{V_2}{V_1} \right) = PV \ln \left( \frac{P_1}{P_2} \right) = m RT \ln \left( \frac{V_2}{V_1} \right) \)

Change in Internal Energy, \( \Delta U = 0 \) (Since \( T_1 = T_2 \))

Heat transfer, \( Q = PV \ln \left( \frac{V_2}{V_1} \right) = PV \ln \left( \frac{P_1}{P_2} \right) = m RT \ln \left( \frac{V_2}{V_1} \right) \)

Change in Enthalpy, \( \Delta H = 0 \) (Since \( T_1 = T_2 \))

Change in Entropy, \( \Delta S = m R \ln \left( \frac{V_2}{V_1} \right) = m R \ln \left( \frac{P_1}{P_2} \right) \)

Note:

For constant temperature process, \( Q = W \)

\( P_1 V_1 = P_2 V_2 \)

(iv) For Reversible Adiabatic process, (or) Isentropic process

\( n = \gamma \)

\( PV^n = C \)

Workdone, \( W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{m R (T_1 - T_2)}{\gamma - 1} \)

Change in Internal Energy, \( \Delta U = m C_v (T_2 - T_1) \)

Heat transfer, \( Q = 0 \)

Change in entropy, \( \Delta S = m C_p (T_2 - T_1) \)

Change in Entropy, \( \Delta S = 0 \) (Since \( s_1 = s_2 \))

Note:

For Isentropic process, \( P_1 V_1^\gamma = P_2 V_2^\gamma \)

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \]

\[ \Delta U = -W \]
(v) For Polytropic process

\[ n = n; \ PV^n = C \]

Workdone,
\[ W = \frac{P_1 V_1 - P_2 V_2}{n - 1} = \frac{mR(T_1 - T_2)}{n - 1} \]

Change in internal Energy,
\[ \Delta U = m \ C_v (T_2 - T_1) \]

Heat transfer,
\[ Q = \gamma \left[ \frac{\gamma - n}{\gamma - 1} \right] W = m \ C_n (T_2 - T_1) \]

Change in Enthalpy,
\[ \Delta H = m \ C_p (T_2 - T_1) \]

Change in Entropy,
\[ \Delta S = m \ C_n \ln \left( \frac{T_2}{T_1} \right) = \frac{\gamma - n}{\gamma - 1} mR \ln \left( \frac{V_2}{V_1} \right) \]

Note:

Here, \( C_n \rightarrow \) Polytropic specific heat

\[ C_n = C_v \left( \frac{\gamma - n}{1 - n} \right) \]

\[ P_1 V_1^n = P_2 V_2^n \]

\[ \frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{\frac{1}{n}} \]

\[ \frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{\frac{n-1}{n}} \]

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \]

First law of Thermodynamics for steady flow process

Steady flow means that the rate of flow of mass and energy are constant.

\[ \dot{m}_1 = \dot{m}_2 = \dot{m} \rightarrow \text{mass flow rate in kg/sec} \]

\[ \rho_1 A_1 C_1 = \rho_2 A_2 C_2 \]

(or)

\[ \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2} \]

Where,

\( \rho \rightarrow \) Density of fluid

\( A \rightarrow \) Area of cross section through which mass flows

\( v \rightarrow \) Specific volume

\( C \rightarrow \) Velocity of fluid
\[ E_1 = E_2 \]

\[ h_1 + \frac{C_1^2}{2} + Z_1 g + Q = h_2 + \frac{C_2^2}{2} + Z_2 g + W \text{ [J/kg]} \]  

...(1)

\[ \dot{m} \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \dot{Q} = \dot{m} \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \dot{W} \text{ [W]} \]  

...(2)

Eqns. (1) and (2) are called steady flow energy equation (S.F.E.E)

Steady flow energy equation for various important devices:

(i) Nozzle

\[ Q = 0 \text{ (Insulated)} \]  

Fig. 1.12
(i) Steady state
(ii) Adiabatic \((Q = 0)\)
(iii) Changes in potential Energy \((P.E) = 0\)
(iv) \(W = 0\)
SFEE becomes,
\[ h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \]
Since \(C_1 << C_2\); \(C_1\) can be neglected.
\[ h_1 = h_2 + \frac{C_2^2}{2} \]
\[ C_2 = \sqrt{2(h_1 - h_2)} \]

(ii) Turbine

Assumptions
(i) Steady state
(ii) Adiabatic \((Q = 0)\)
(iii) \(\Delta KE \& \Delta PE = 0\)
SFEE becomes,
\[ h_1 = h_2 + W \Rightarrow W = h_1 - h_2 \]
[Workdone by system (+ ve)]
(iii) **Compressor**

**Assumptions**

(i) Steady state

(ii) Adiabatic \((Q = 0)\)

(iii) \(\Delta PE \& \Delta KE = 0\)

\[ h_1 = h_2 - W \Rightarrow W = h_2 - h_1 \]

[Work is done on the system (−ve W)]

(iv) **Throttling**

**Assumptions**

(i) Steady state

(ii) Adiabatic \((Q = 0)\)

(iii) \(\Delta KE \& \Delta PE = 0\)

(iv) \(W = 0\)

\[ h_1 = h_2 \] (Isenthalpic Process)
(v) Heat Exchanger

Assumptions

(i) Steady state
(ii) Adiabatic \( (Q = 0) \)
(iii) \( \Delta PE \ & \ \Delta KE = 0 \)
(iv) \( W = 0 \)

\[ \dot{m}_h \left( h_1 - h_2 \right) = \dot{m}_c \left( h_2' - h_1' \right) \]

Important University Question Bank

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